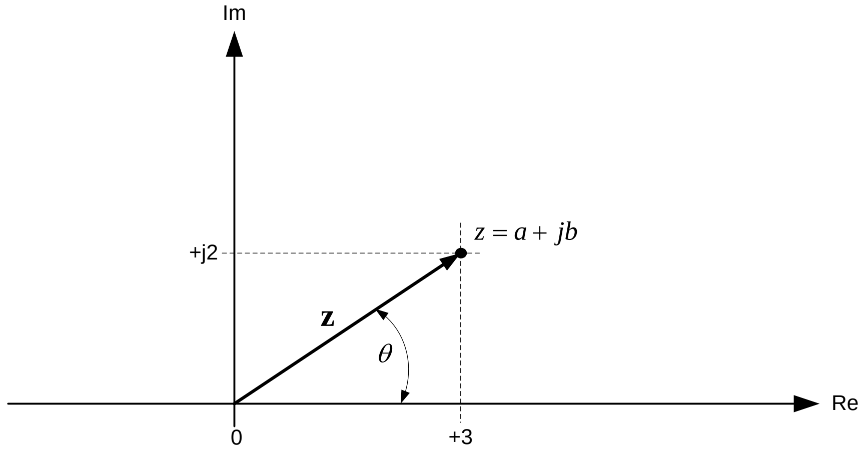
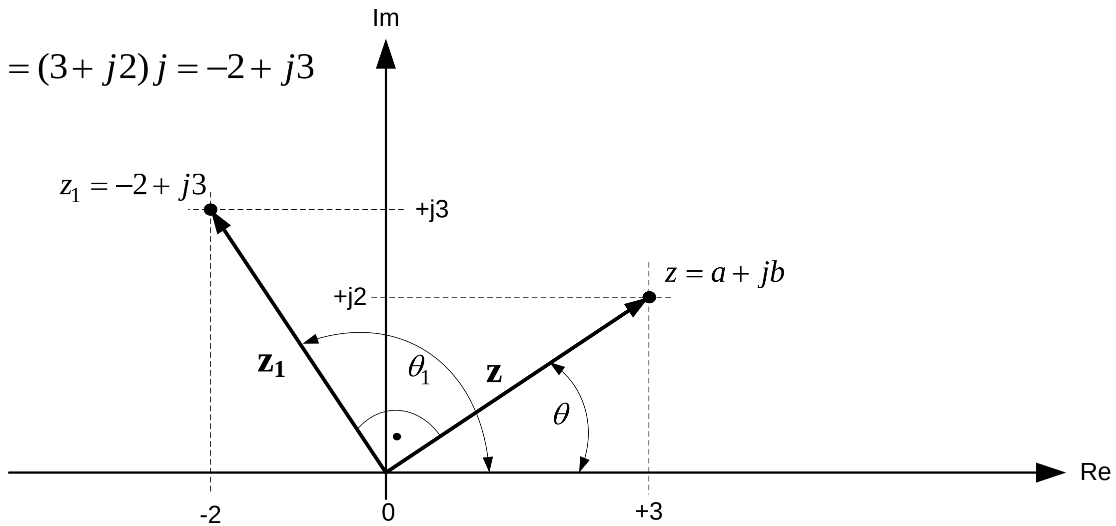


1.a graphical representation of complex numbers

$$z_1 = z.j = (3 + j2)\,j = -2 + j3$$



1.b multiplying of complex number by $+j$ operator

1.c complex number representation:

Cartesian: $z = a + jb$ coordinates in complex plane: (a, b)

Polar: $z = \mathbf{z}\left(\cos\theta + j\sin\theta\right)$

coordinates in complex plane : $\left(\mathbf{z}, \theta\right)$

components:

$\mathrm{Re} = \mathbf{z}\cos\theta$ a $\mathrm{Im} = j\mathbf{z}\sin\theta$

Exponential: $z = \mathbf{z}\,e^{j\theta}$

Euler formula:

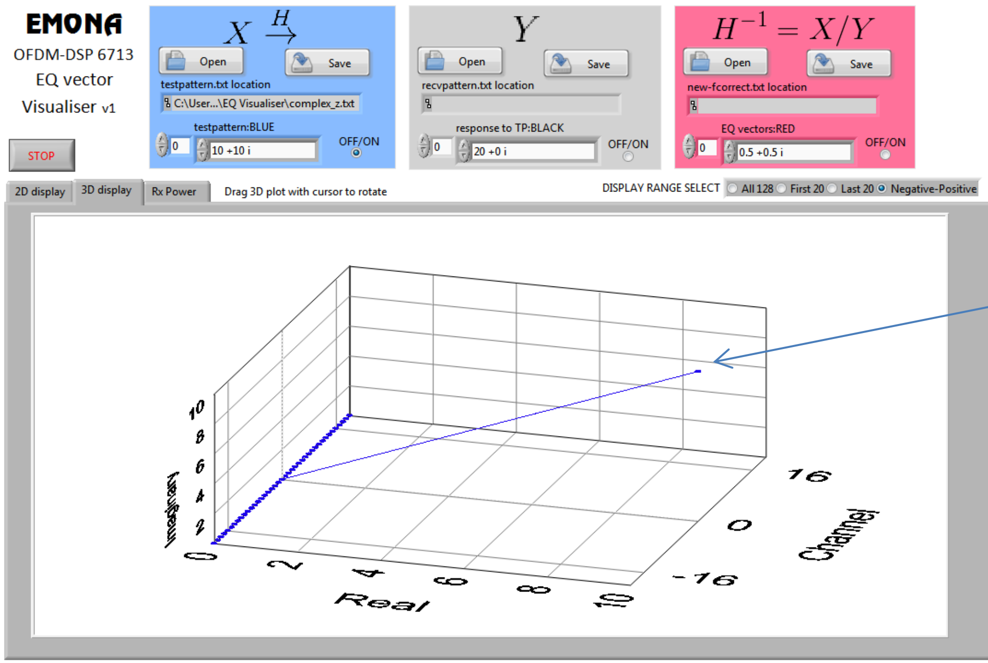
$$e^{j\theta} = \cos\theta + j\sin\theta$$

magnitude:

$$\mathbf{z} = \left|z\right| = \sqrt{a^2 + b^2}$$

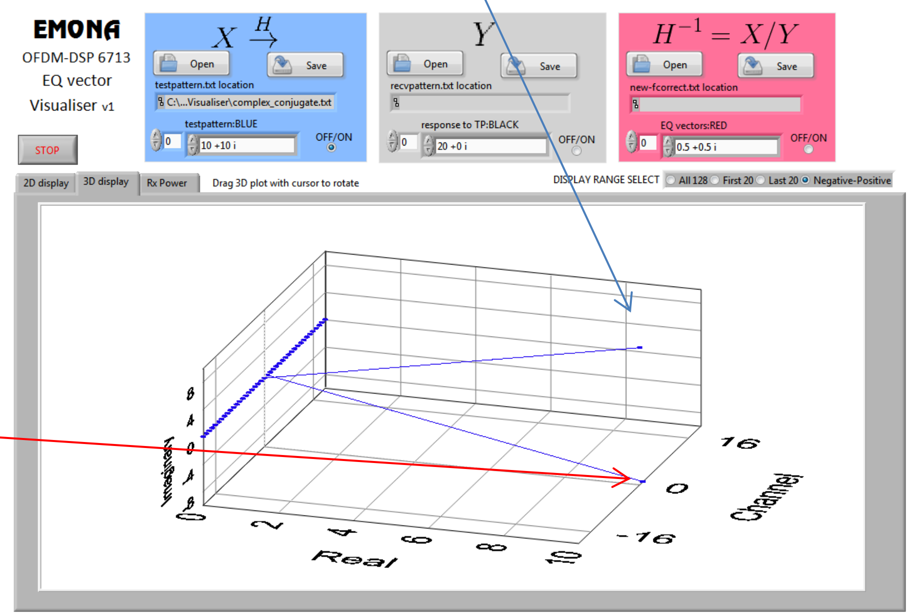
phase (argument):

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$



$$z = 10 + j10$$

$$z = 10 - j10$$



Derivation of Euler formula:

Taylor expansion of an exponential function: $e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \frac{z^5}{5!} + \frac{z^6}{6!} + \cdots$

for z we put: $j\theta$ then:

$$e^{j\theta} = 1 + j\theta + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \frac{(j\theta)^5}{5!} + \frac{(j\theta)^6}{6!} + \cdots$$
$$= 1 + j\theta - \frac{\theta^2}{2!} - j\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + j\frac{\theta^5}{5!} - \frac{\theta^6}{6!} + \cdots$$

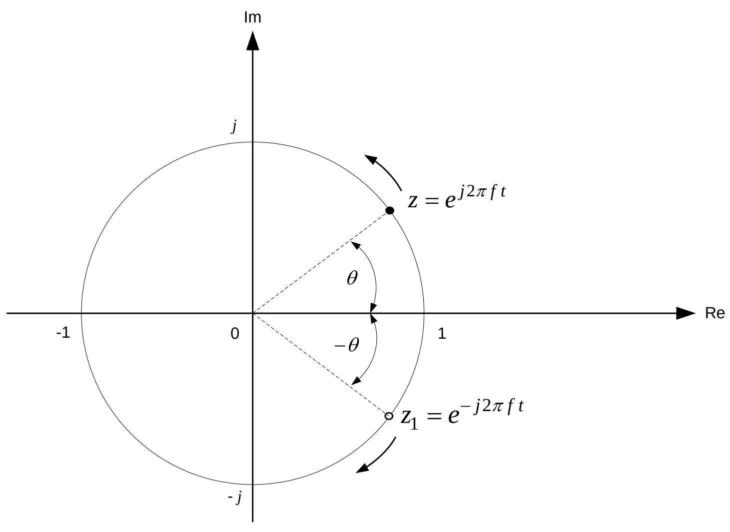
similarly:

$$e^{-j\theta} = 1 - j\theta + \frac{(-j\theta)^2}{2!} + \frac{(-j\theta)^3}{3!} + \frac{(-j\theta)^4}{4!} + \frac{(-j\theta)^5}{5!} + \frac{(-j\theta)^6}{6!} + \cdots$$
$$= 1 - j\theta - \frac{\theta^2}{2!} + j\frac{\theta^3}{3!} + \frac{\theta^4}{4!} - j\frac{\theta^5}{5!} - \frac{\theta^6}{6!} + \cdots$$

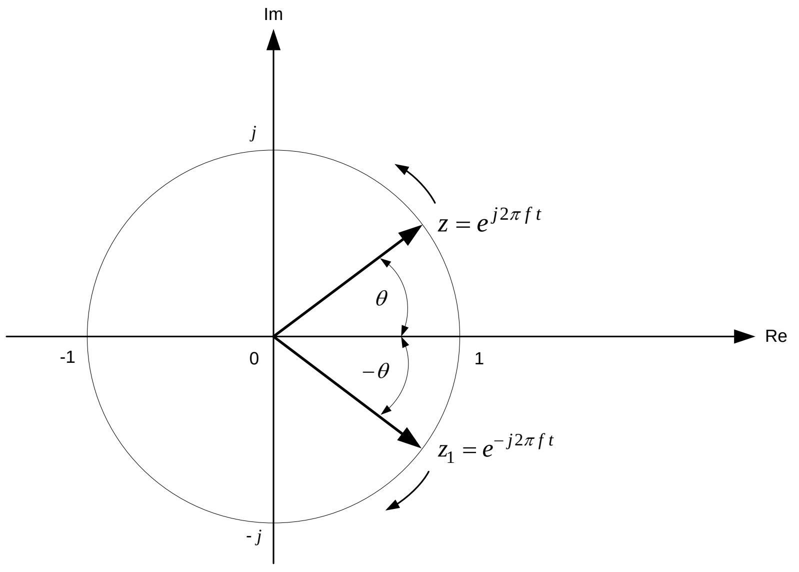
by separating real and imaginary parts and combining: $e^{j\theta}$ and $e^{-j\theta}$ we get:

$$\mathrm{Re}\left(e^{j\theta}\right) = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots = \frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta$$

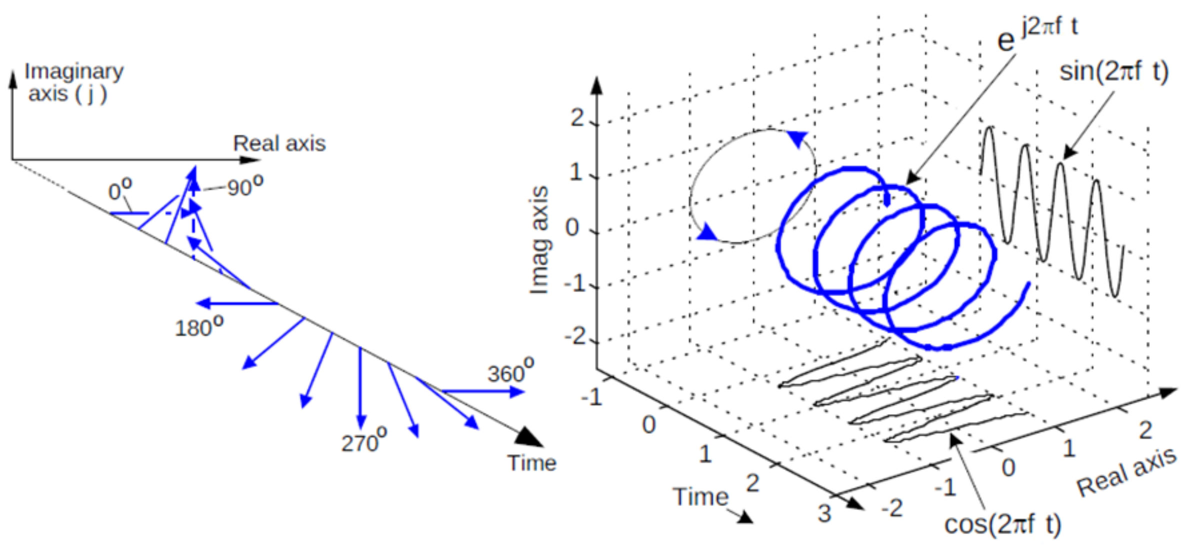
$$\mathrm{Im}\left(e^{j\theta}\right) = j\theta - j\frac{\theta^3}{3!} + j\frac{\theta^5}{5!} - \cdots = \frac{e^{j\theta} - e^{-j\theta}}{2j} = \sin\theta$$



a. Complex number in exponential form

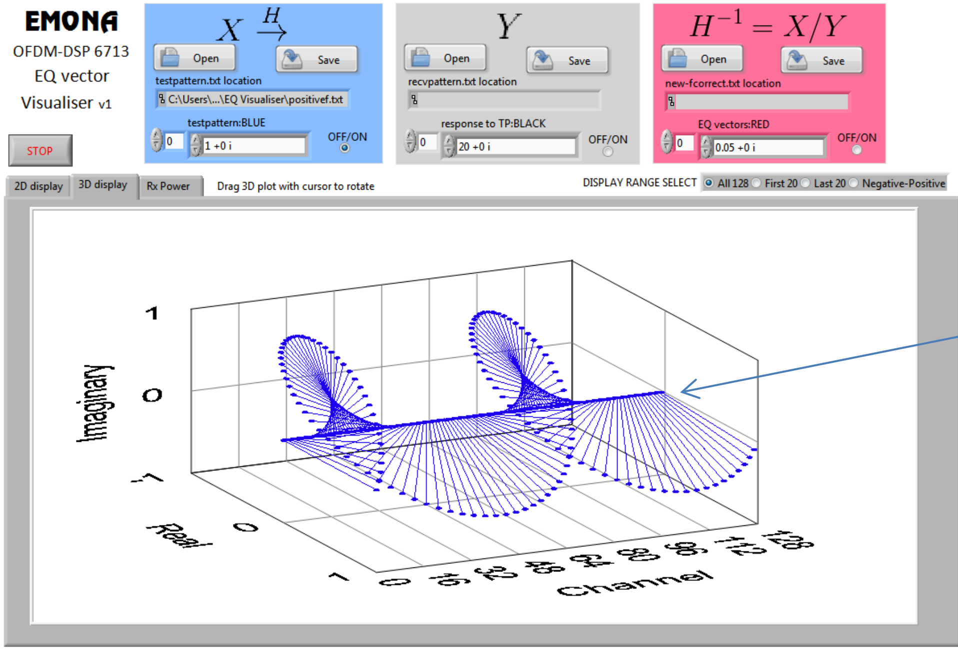


b. Rotating phasor

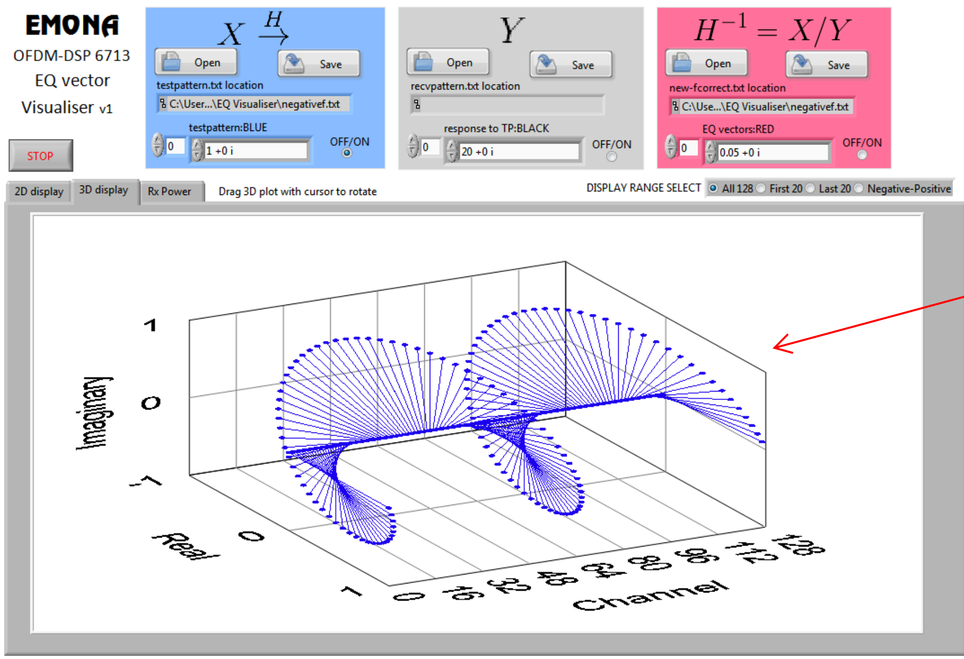


c. Phasor rotation in time

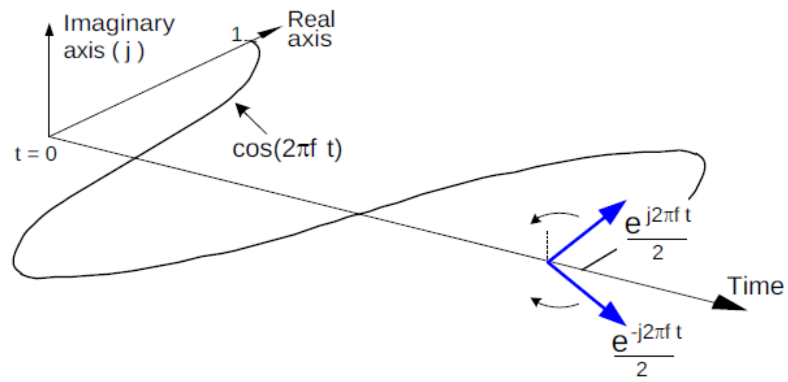
Source: Lyons, R., Quadrature Signals: Complex, But Not Complicated



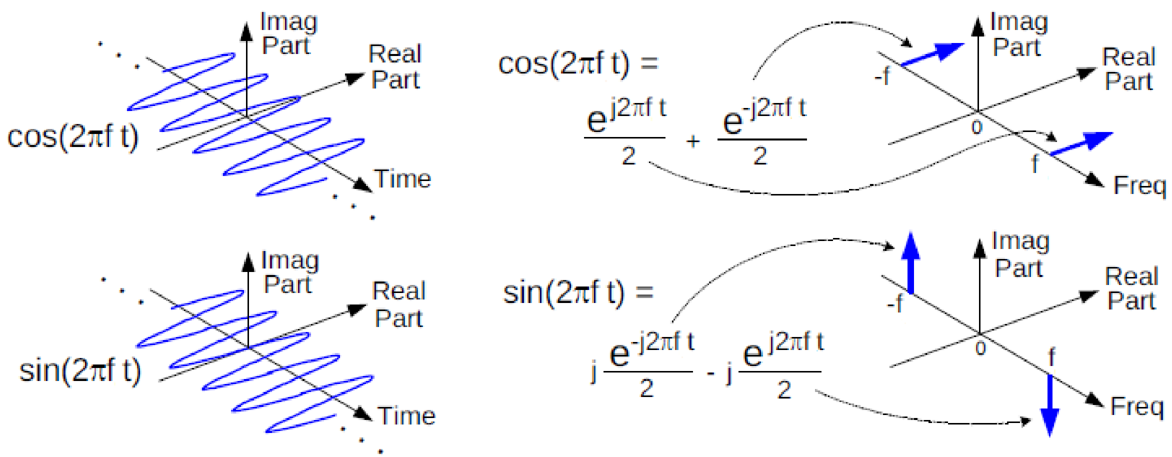
$$z = e^{j8\pi f t}$$



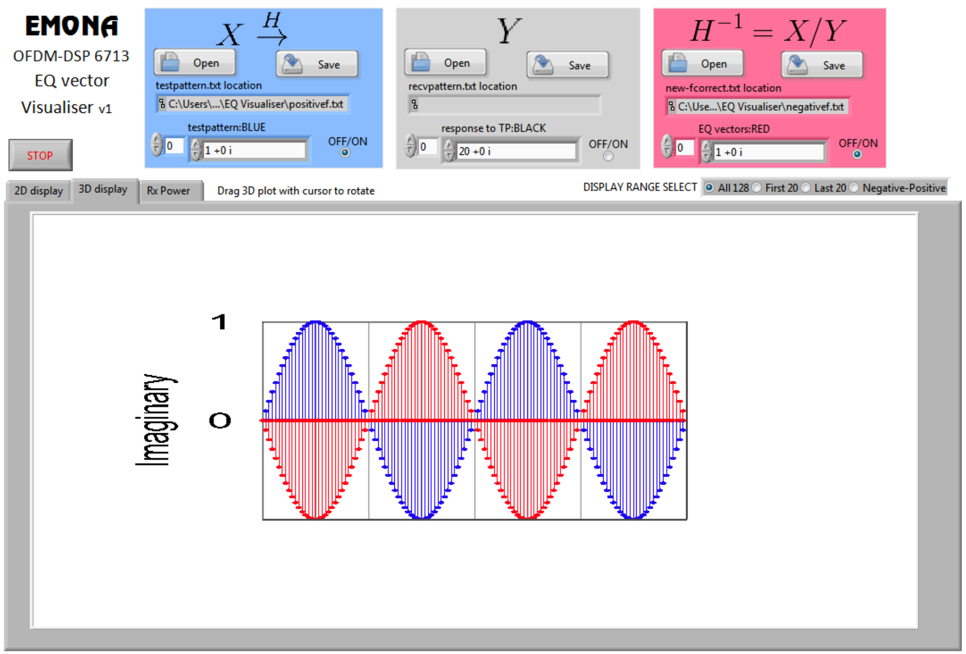
$$z^{*} = e^{-j8\pi f t}$$



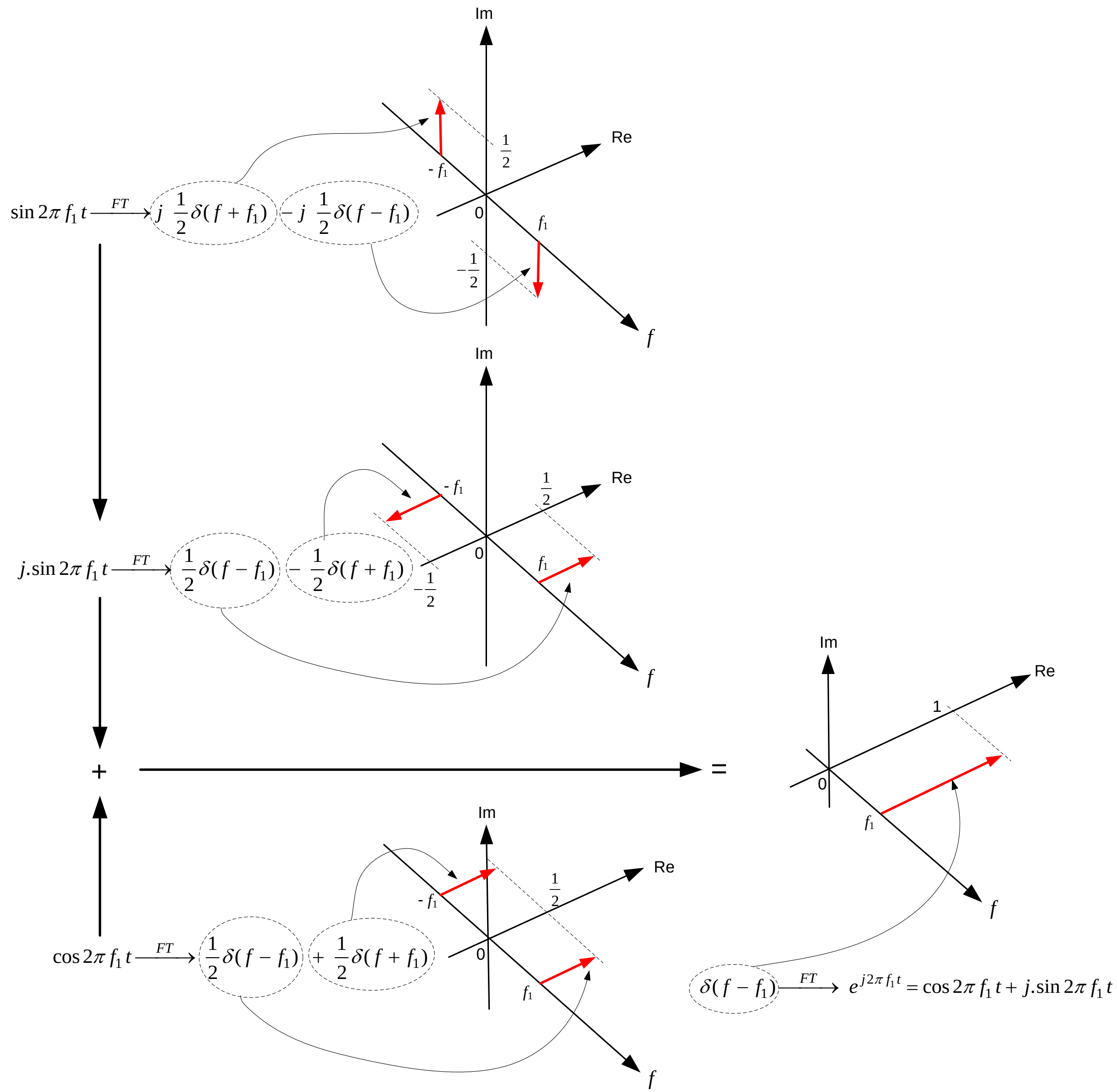
a. The sum of two oppositely rotating phasors
Source: Lyons, R., Quadrature Signals: Complex, But Not Complicated



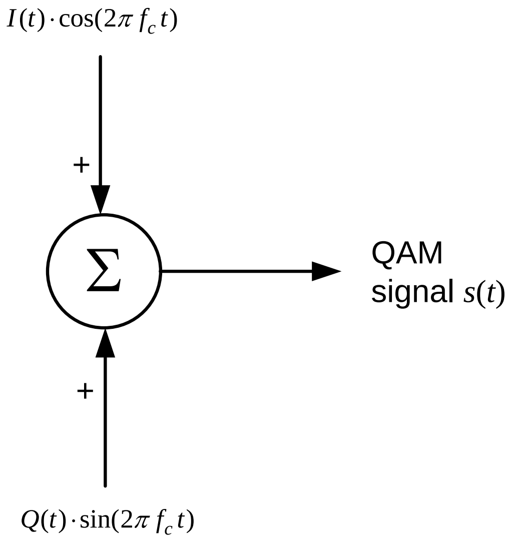
a. Sin and Cos in frequency domain
Source: Lyons, R., Quadrature Signals: Complex, But Not Complicated



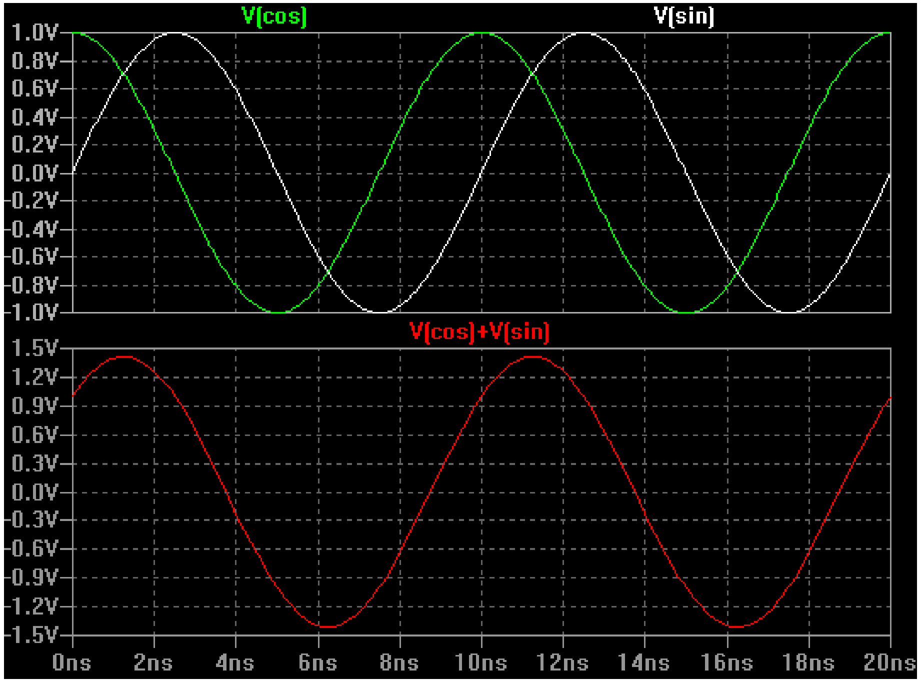
$$z = e^{j8\pi f t} + e^{-j8\pi f t}$$



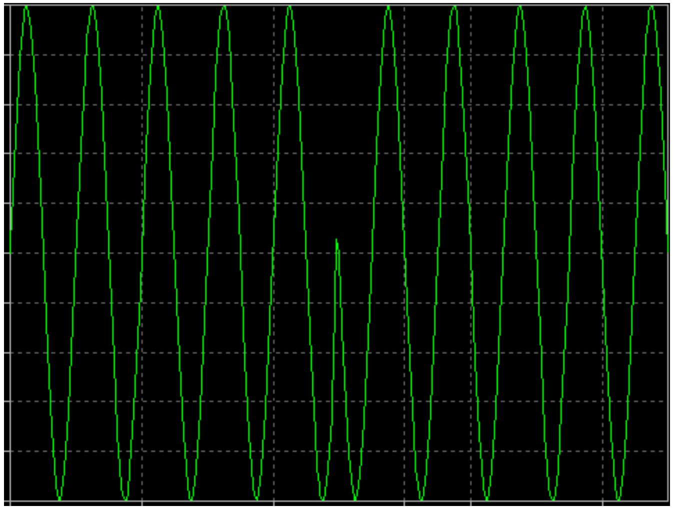
Graphical proof of Euler formula



a. Generation of QAM signal



b. sum of quadrature signals
Source: http://www.tek.com/blog/what's-your-iq---about-quadrature-signals...



c. BPSK modulation
Source: http://www.tek.com/blog/what's-your-iq---about-quadrature-signals...

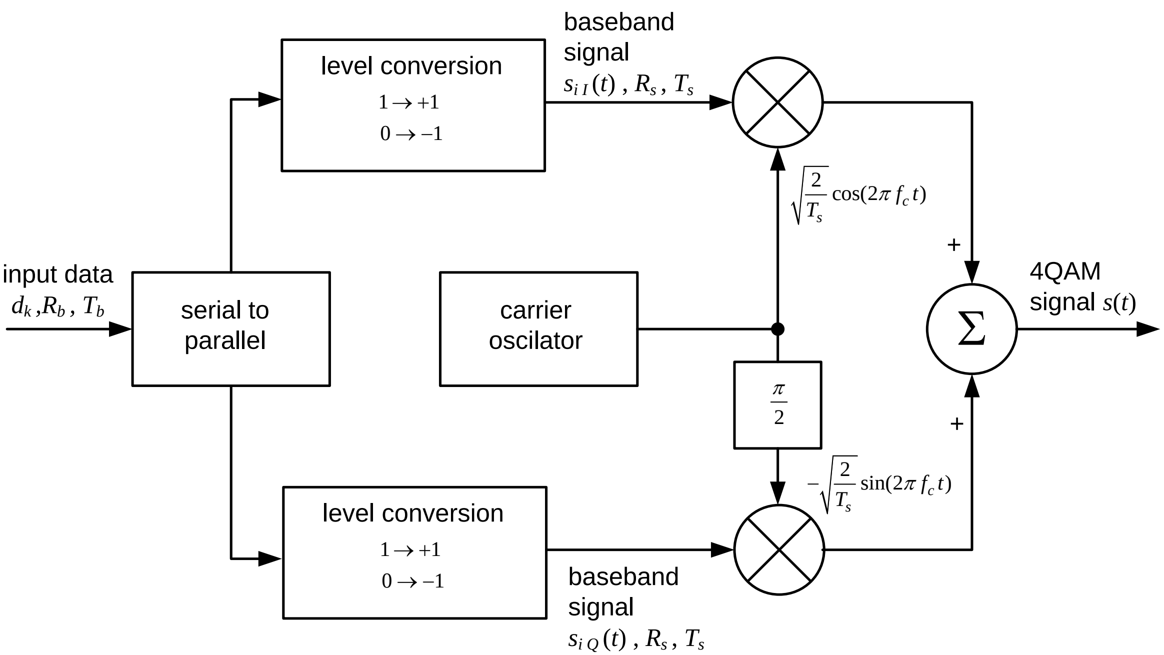
$$I(t) = +1 \wedge Q(t) = +1 \; ; \; 0 \le t \le T_s \quad \Rightarrow \theta = 45^\circ$$

$$I(t) = -1 \wedge Q(t) = +1 \; ; \; 0 \le t \le T_s \quad \Rightarrow \theta = 135^\circ$$

$$I(t) = -1 \wedge Q(t) = -1 \; ; \; 0 \le t \le T_s \quad \Rightarrow \theta = 225^\circ$$

$$I(t) = +1 \wedge Q(t) = -1 \; ; \; 0 \le t \le T_s \quad \Rightarrow \theta = 315^\circ$$

a. Phases of QPSK signal



b. 4QAM modulator

Analytical form of M-QAM signal

$$s_i(t) = s_{i\_I}\psi_1(t) + s_{i\_Q}\psi_2(t) \qquad i = 1,\ldots,M$$

$$\psi_1(t) = \sqrt{\frac{2}{E_p}}\,p(t)\cos\left(2\pi f_c t\right), \qquad 0 \le t \le T_s$$

$$\psi_2(t) = -\sqrt{\frac{2}{E_p}}\,p(t)\sin\left(2\pi f_c t\right), \qquad 0 \le t \le T_s$$

$$s_{i\_I} = \sqrt{\frac{E_p}{2}}A_{i\_I} = \sqrt{\frac{E_p}{2}}A_i\cos\theta_i$$

$$s_{i\_I} = \sqrt{\frac{E_p}{2}}A_{i\_Q} = \sqrt{\frac{E_p}{2}}A_i\sin\theta_i$$

$$E_{avg} = \frac{1}{2}E_p E\left\{A_i^2\right\}$$ Average signal energy

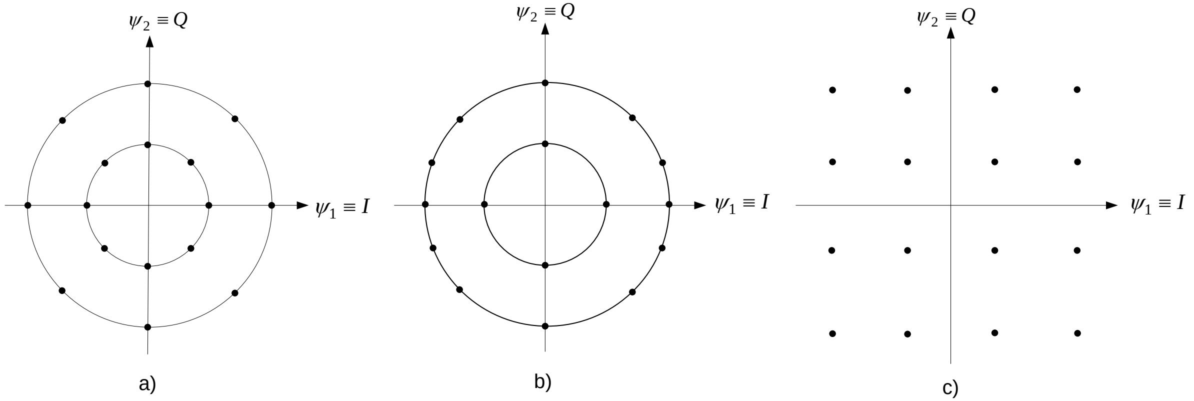
$$P_{avg} = \frac{E_{avg}}{T_s}$$ Average signal power

$$A_{avg} = \sqrt{2P_{avg}}$$ Average signal amplitude

$$\boldsymbol{s}_i = \left(s_{i\_I}, s_{i\_Q}\right) \quad i = 1,\ldots,M$$ Phasor form of M-QAM signal

$$\left\|\boldsymbol{s}_i\right\| = \sqrt{s_{i\_I}^2 + s_{i\_Q}^2} = \sqrt{E_i}$$ Magnitude of signal phasor

$$d_{ij} = \sqrt{\left|\boldsymbol{s}_i - \boldsymbol{s}_j\right|^2} = \sqrt{\left(s_{i\_I} - s_{j\_I}\right)^2 + \left(s_{i\_Q} - s_{j\_Q}\right)^2}, \quad i, j = 1, 2, \ldots, M$$ Distance between arbitrary pair of phasors



a. Types of M-QAM constellations

b. analytical form of the M-QAM signal for a square constellation

$$s_i(t) = I_i \sqrt{\frac{E_0}{E_p}}\psi_1(t) + Q_i \sqrt{\frac{E_0}{E_p}}\psi_2(t)$$

$$\left[I_i, Q_i\right] = \begin{bmatrix} (-L+1, L-1) & (-L+3, L-1) & \cdots & (L-1, L-1) \\ (-L+1, L-3) & (-L+3, L-3) & \cdots & (L-1, L-3) \\ \vdots & \vdots & & \vdots \\ (-L+1, -L+1) & (-L+3, -L+1) & \cdots & (L-1, -L+1) \end{bmatrix}$$

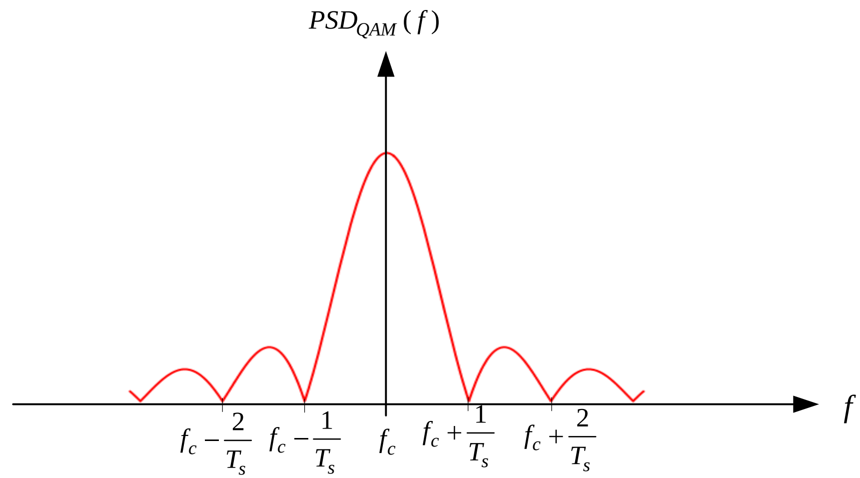
$$L = \sqrt{M}, \ M = 4^n, \ n = 1, 2, 3, \ldots$$

c. M-QAM constellation matrix

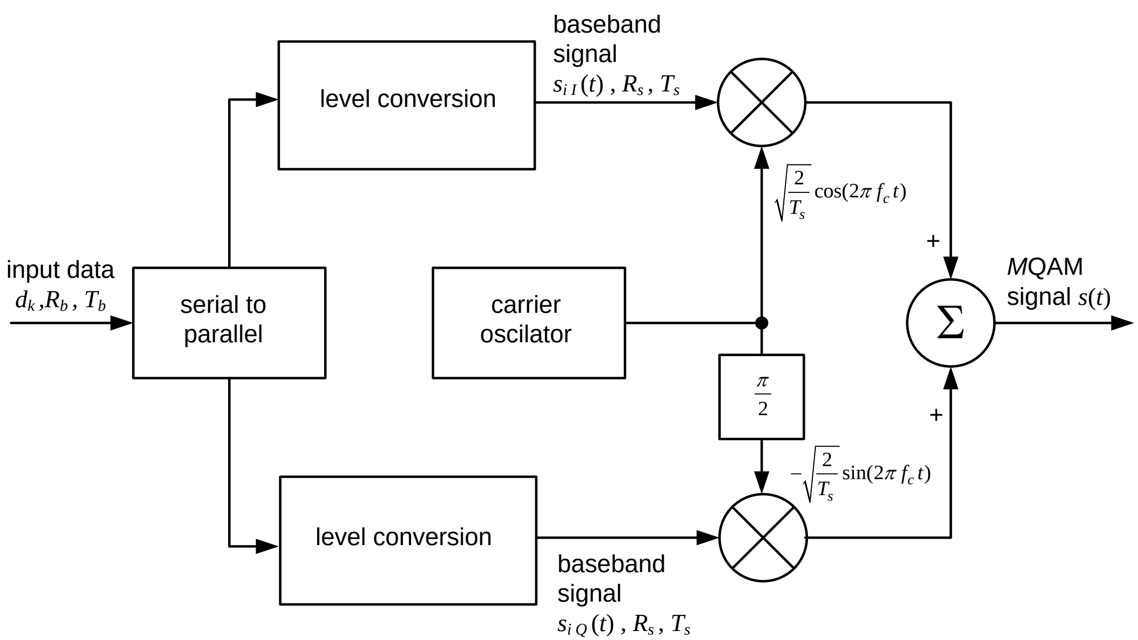
$$\left[I_i, Q_i\right] = \begin{bmatrix} (-3,3) & (-1,3) & (1,3) & (3,3) \\ (-3,1) & (-1,1) & (1,1) & (3,1) \\ (-3,-1) & (-1,-1) & (1,-1) & (3,-1) \\ (-3,-3) & (-1,-3) & (1,-3) & (3,-3) \end{bmatrix}$$

d. constellation matrix of 16QAM

$$PSD_{QAM}(f) = \frac{2E_0}{3}(M-1)\left(\frac{\sin(\pi f \, kT_b)}{\pi f \, kT_b}\right)^2$$



a. PSD of square M-QAM



b. MQAM modulator

$$N = 128$$

$$T_{sample\_actual} = \frac{1[ms]}{N} = \frac{10^{-3}}{128} = 7.8125\ [\mu s]$$

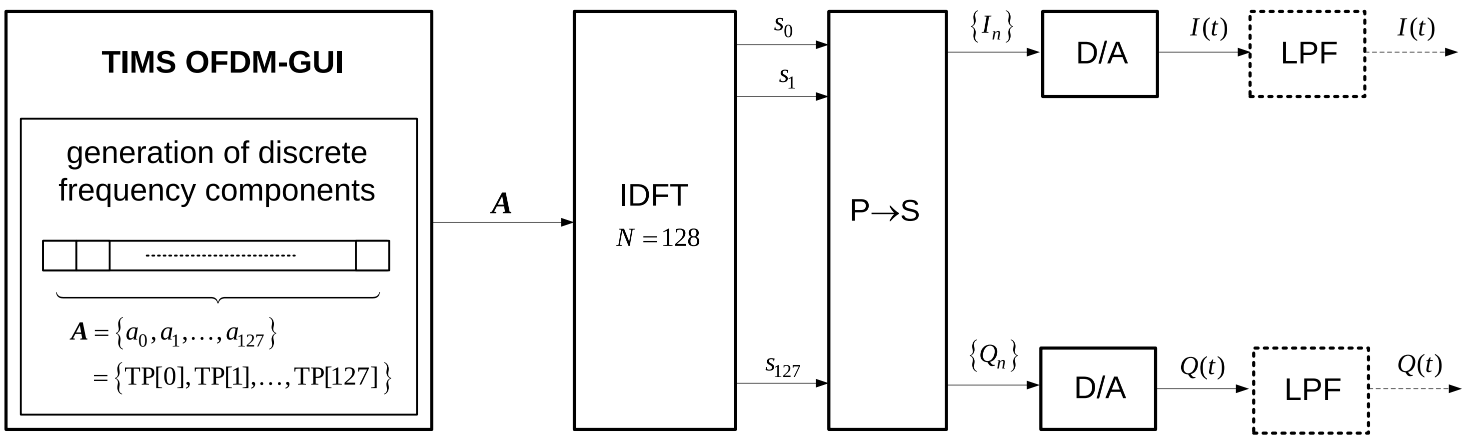
$$f_u = \frac{1}{N \cdot \frac{1}{T_{sample\_actual}}} = \frac{1}{128 \times \frac{1[ms]}{128}} = 1\ [kHz]$$

$$\boldsymbol{A} = \{a_0, a_1, \ldots, a_{127}\} = \{\mathrm{TP}[0], \mathrm{TP}[1], \ldots, \mathrm{TP}[127]\}$$

$$s_k = \frac{1}{N}\sum_{n=0}^{N-1} a_n \exp\left[\frac{j2\pi kn}{N}\right], \qquad k = 0, 1, \ldots, N-1$$

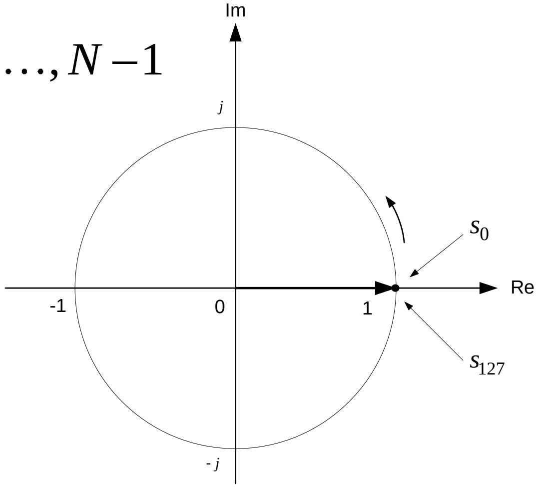
$$s_k = \sum_{n=0}^{N-1} a_n \exp\left[\frac{j2\pi kn}{N}\right], \quad k = 0, 1, \ldots, N-1$$

a. Basic parameters of TIMS IDFT



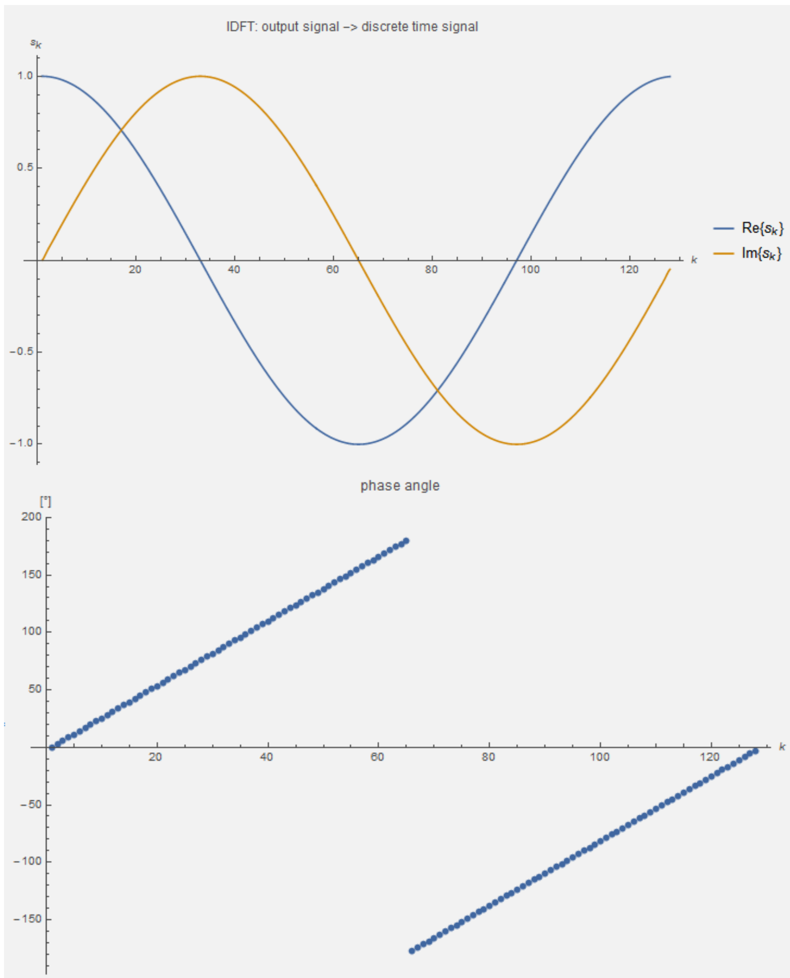
b. Block diagram of TIMS IDFT

a. DC component: $a_0 \neq 0 \quad \wedge \quad a_1, a_2, \ldots, a_{127} = 0 \qquad s_k = a_0 \,, \quad k = 0,1,\ldots, N-1$



b. The lowest frequency:

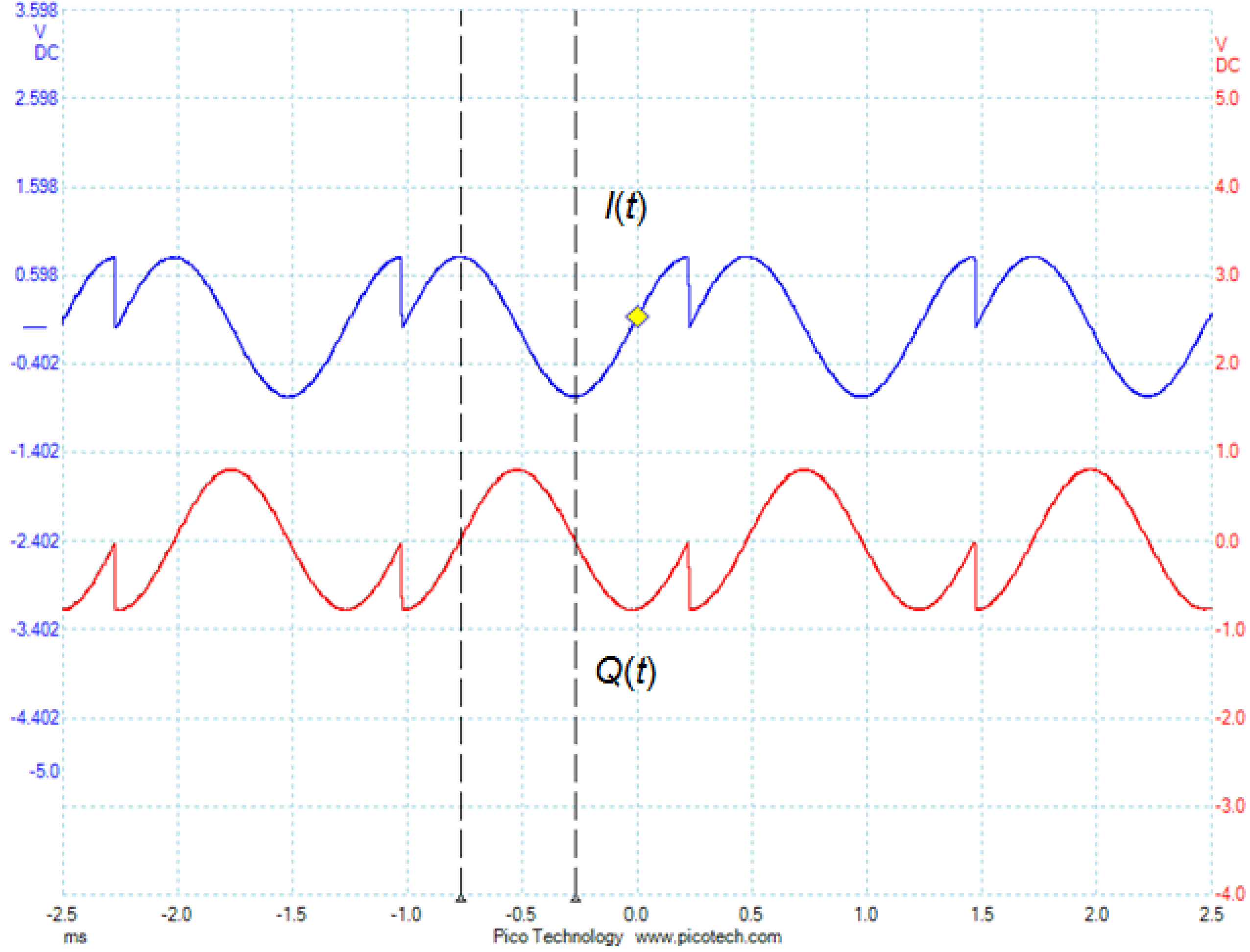
$$a_1 \neq 0 \quad \wedge \quad a_0, a_2, \ldots, a_{127} = 0 \qquad s_k = a_1 \exp\left[\frac{j2\pi k}{N}\right], \qquad k = 0,1,\ldots, N-1$$



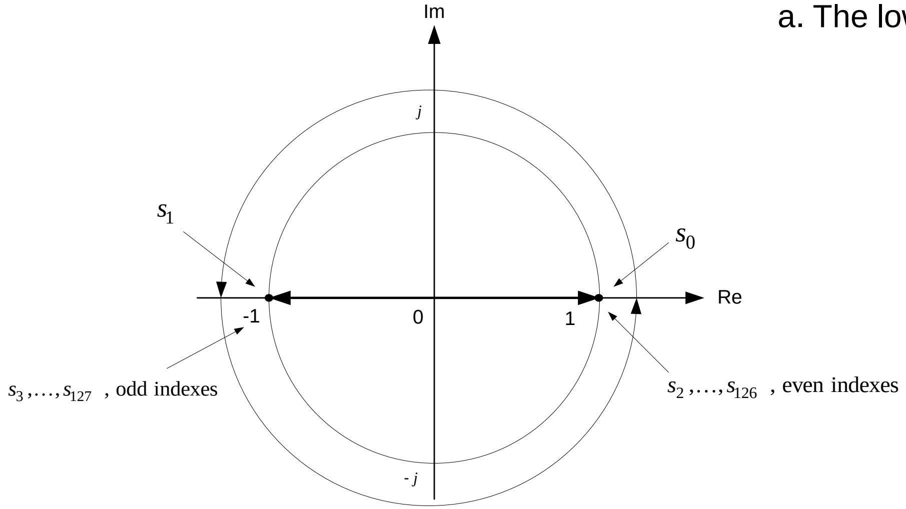
c. IDFT fundamental frequency:

$$f_u = 1\,[kHz] \qquad a_1 = 1$$

d. Phase of the output IDFT signal



The lowest frequency : f = 1 kHz



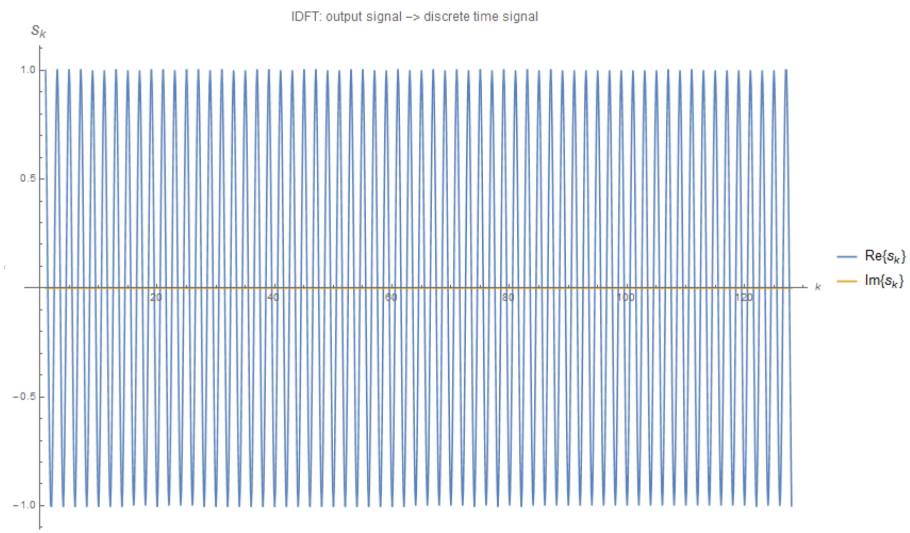
a. The lowest frequency in complex plane

$a_{N/2} \neq 0$ the rest of coordinates = 0

$$s_k = a_{N/2}\exp\left[j\pi k\right], \qquad k = 0,1,\ldots,N-1$$

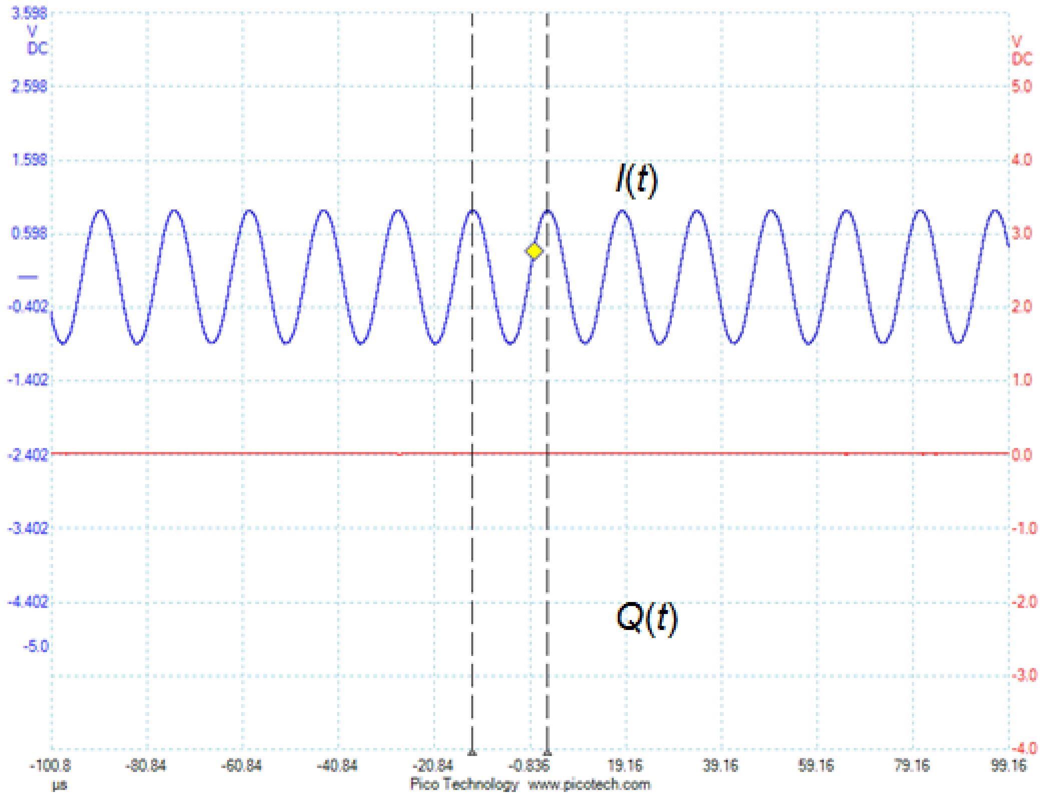
$$s_k = \pm\, a_{N/2}\,, \qquad k = 0,1,\ldots,N-1$$

$$f_{\max} = \frac{1}{2.t_{sample\_actual}} = \frac{1}{2\times\frac{1[ms]}{128}} = 64\ [kHz]$$



b. IDFT the highest frequency: $f_{\max} = 64\ [kHz]$ $a_{64} = 1$

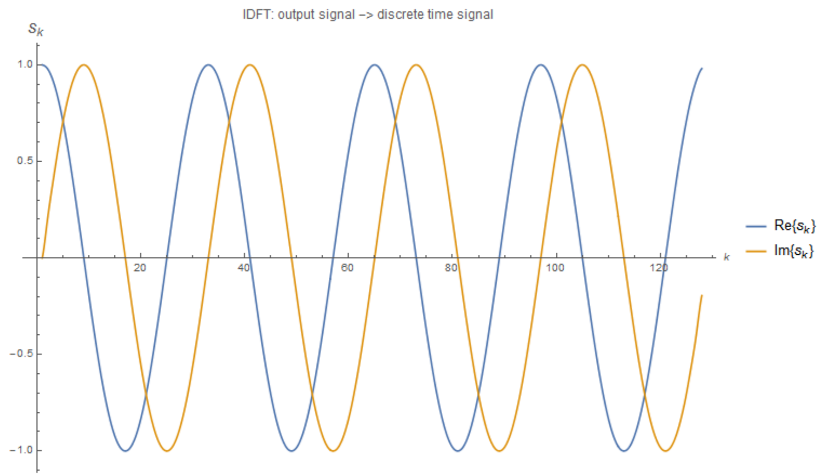
c. Phase of the output IDFT signal



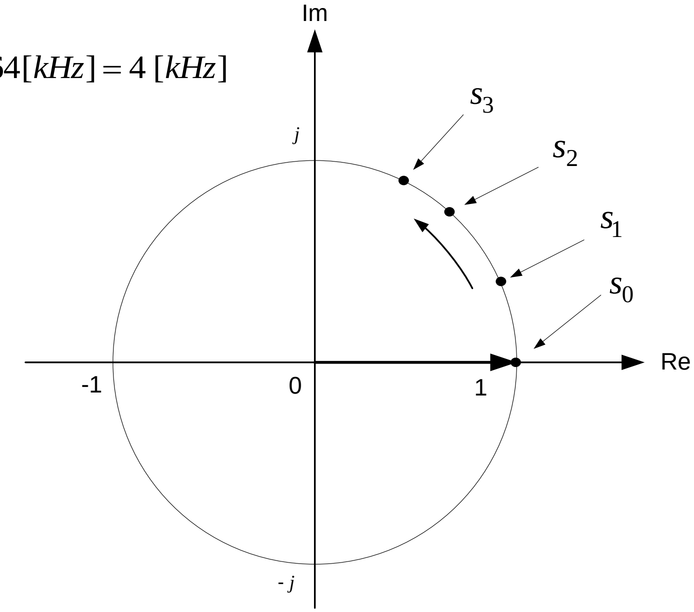
the highest frequency : f_{max}= 64 kHz

a. Positive frequencies: $a_i \neq 0, i \in \langle 1, N/2 \rangle$ $\quad f_k = \frac{k}{N} \cdot 2 f_s, \quad k = 1, 2, \ldots, N/2$

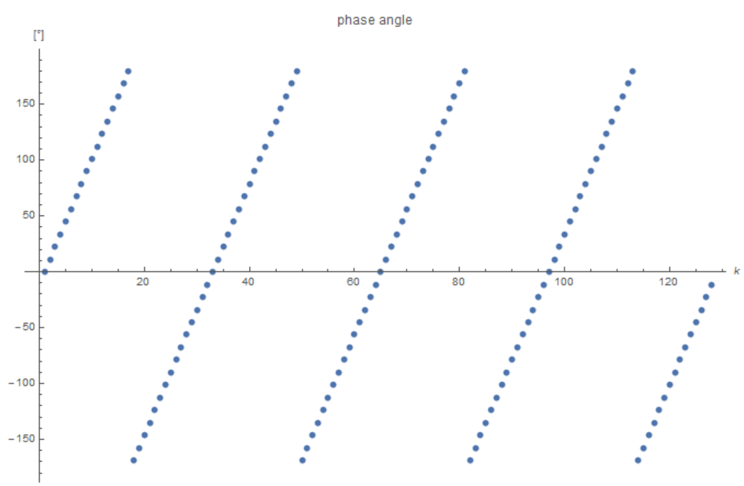
let: $a_4 \neq 0 \wedge a_4 > 0$ the rest of coordinates = 0 $\quad f_4 = \frac{4}{128} \times 2 \times 64 [kHz] = 4\ [kHz]$



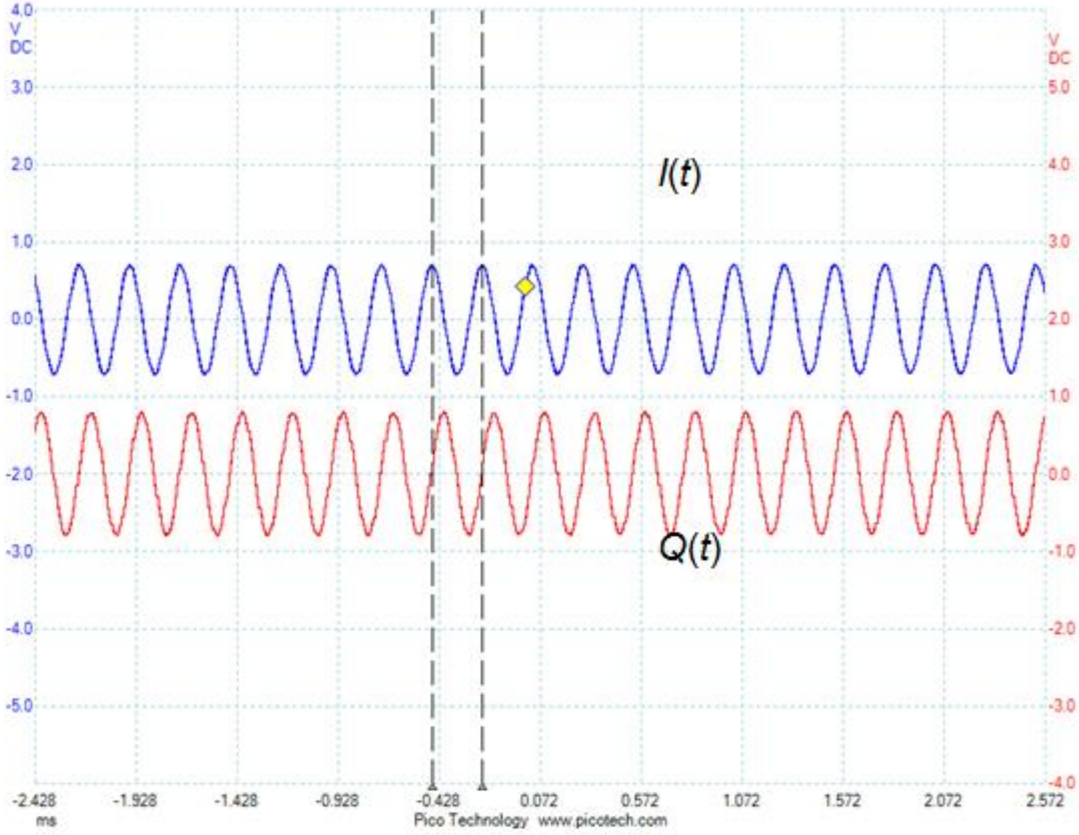
c. Positive frequency : $f_4 = 4\ [kHz]$ $\quad a_4 = 1$



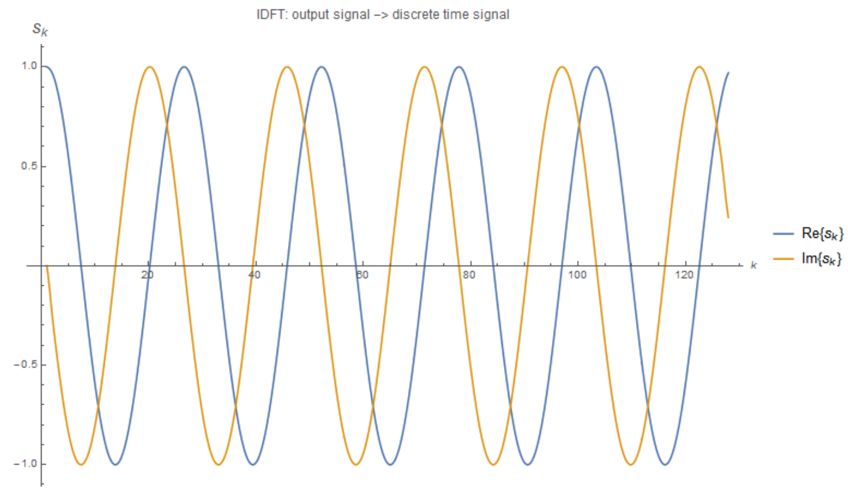
b. Positive frequencies in complex plane



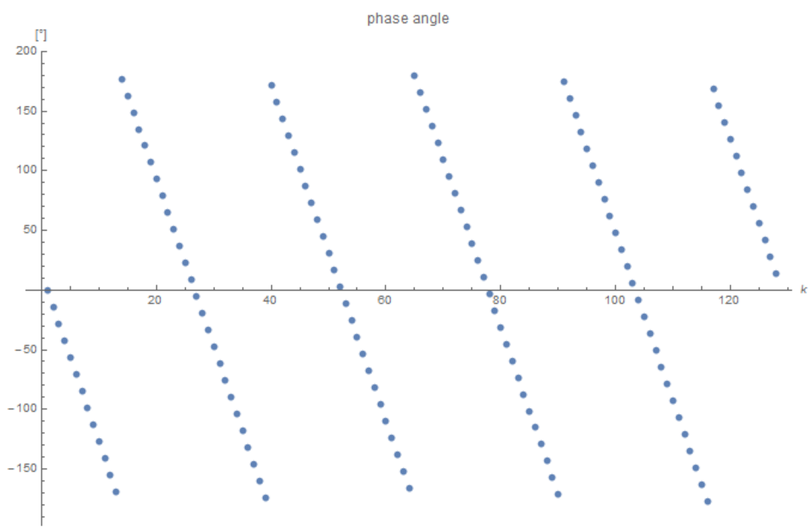
d. Phase of the output IDFT signal



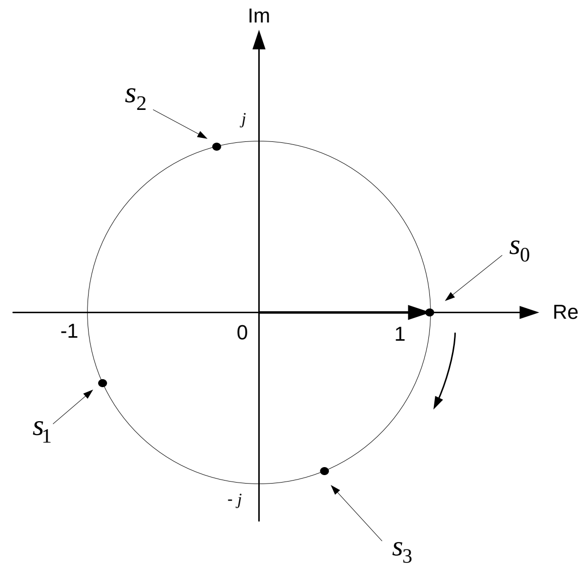
Positive frequency : f = 4 kHz



b. Negative frequency: $f_{124} = -4\ [kHz]$ $\quad a_{124} = 1$



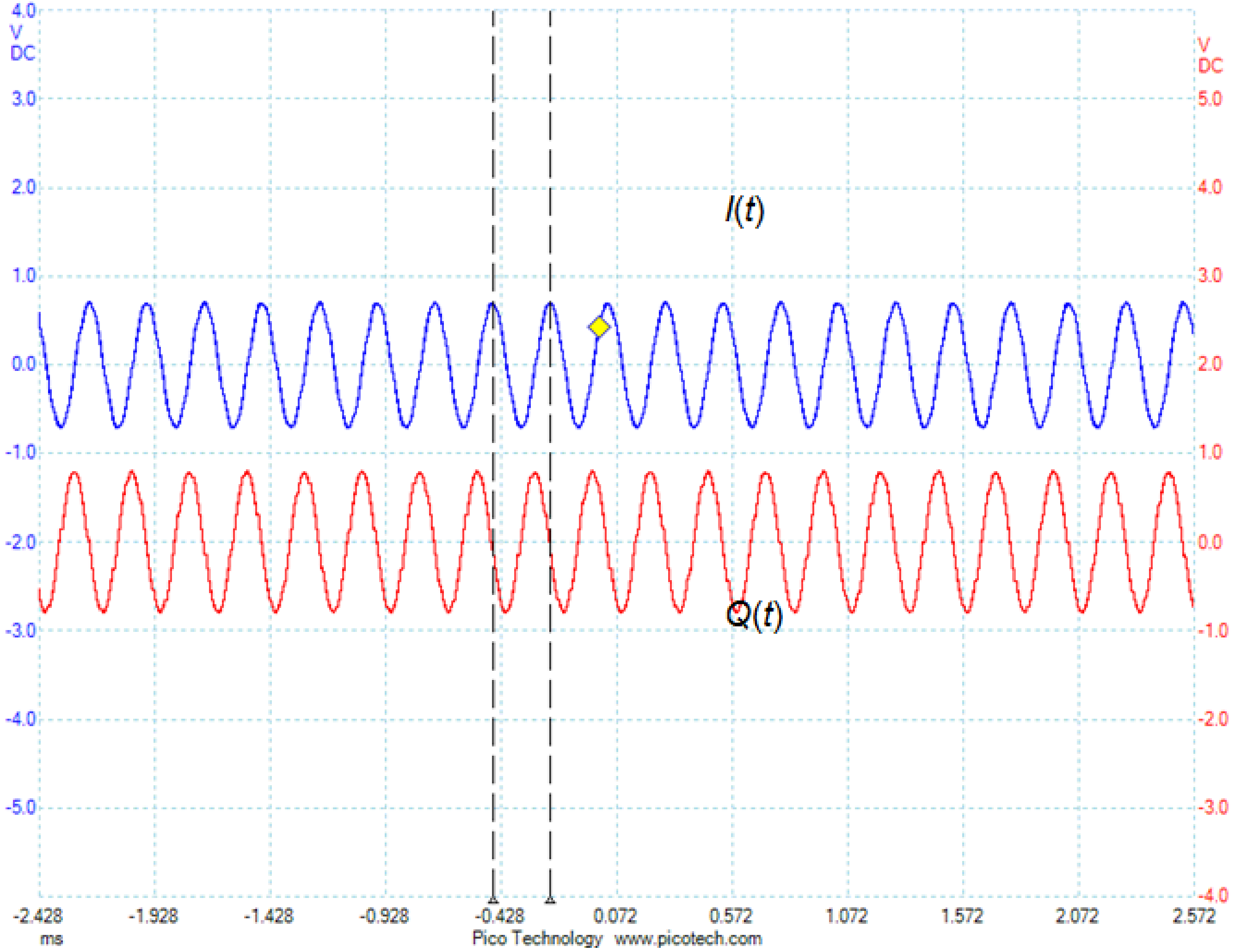
c. Phase of the output IDFT signal



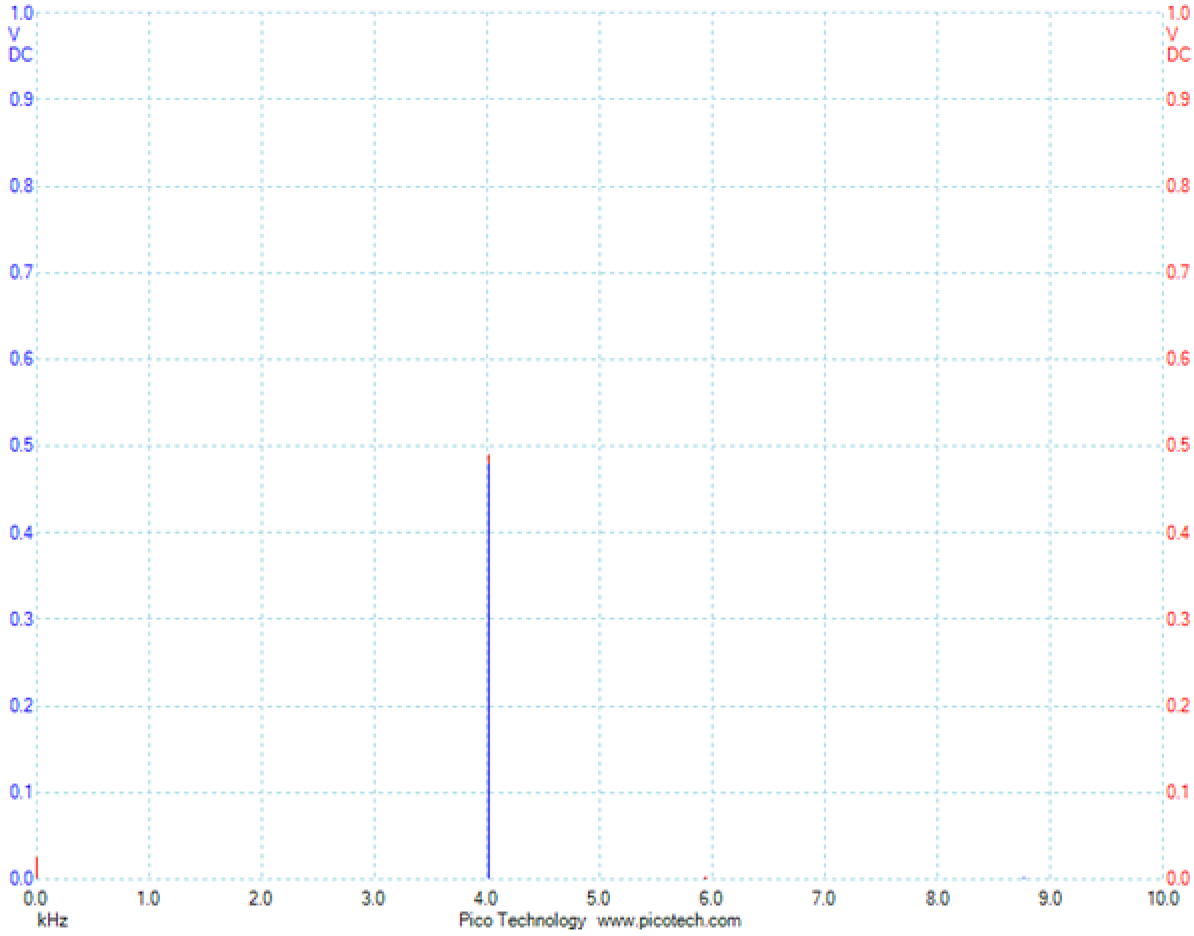
a. Negative frequencies : $a_i \neq 0,\ i \in \left(N/2, N-1 \right\rangle$

frekvencie: $-(f_{\max} - f_u) \ \rightarrow \ -f_u$

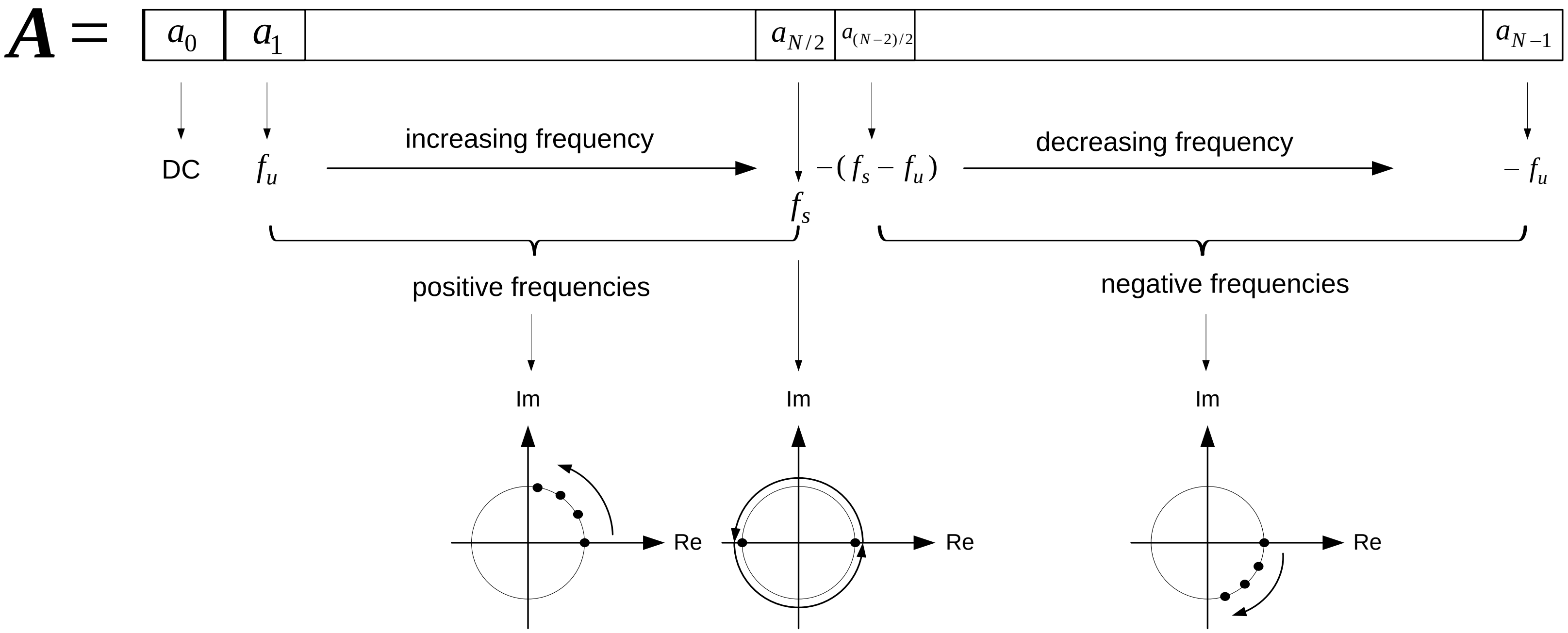
let: $a_{124} \neq 0 \wedge a_{124} > 0$ the rest of coordinates = 0



Negative frequency : f = - 4 kHz



Negative frequency: f = - 4 kHz



a. IDFT output summary

$$s_{1\_k} = a_1 \exp\left[j \frac{2\pi k}{N} \right], \qquad k - 0,1,\ldots,N-1$$

$$s_{2\_k} = a_1 \exp\left[j \frac{2\pi k(N-1)}{N} \right], \qquad k - 0,1,\ldots,N-1$$

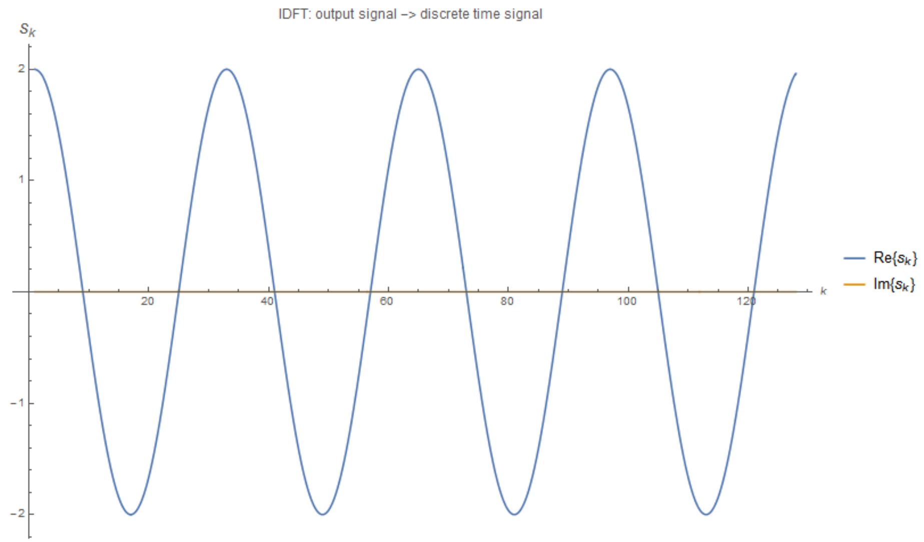
$$s_{1\_k} + s_{2\_k} = a_1 \exp\left[j \frac{2\pi k}{N} \right] + a_1 \exp\left[j \frac{2\pi k(N-1)}{N} \right]$$

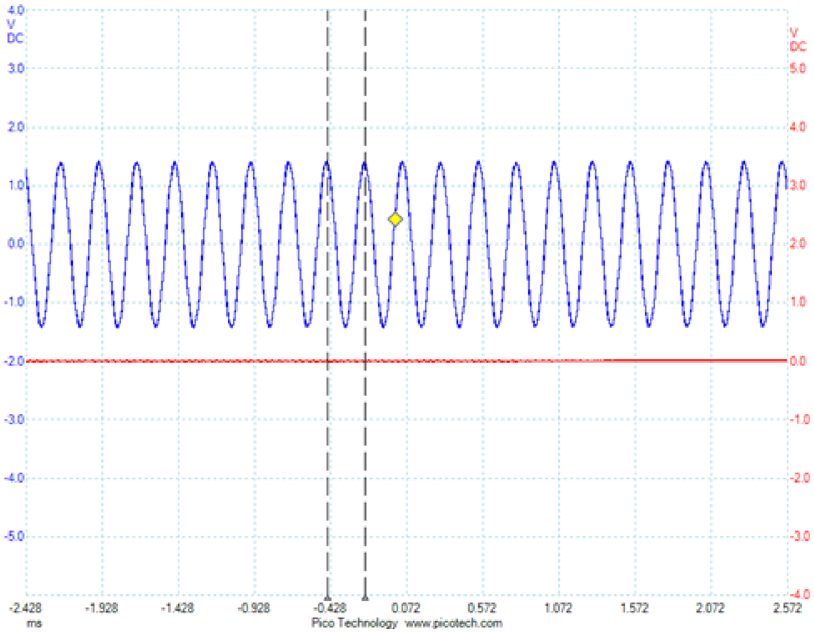
$$= a_1 \exp\left[j \frac{2\pi k}{N} \right] + a_1 \exp\left[-j \frac{2\pi k}{N} \right]$$

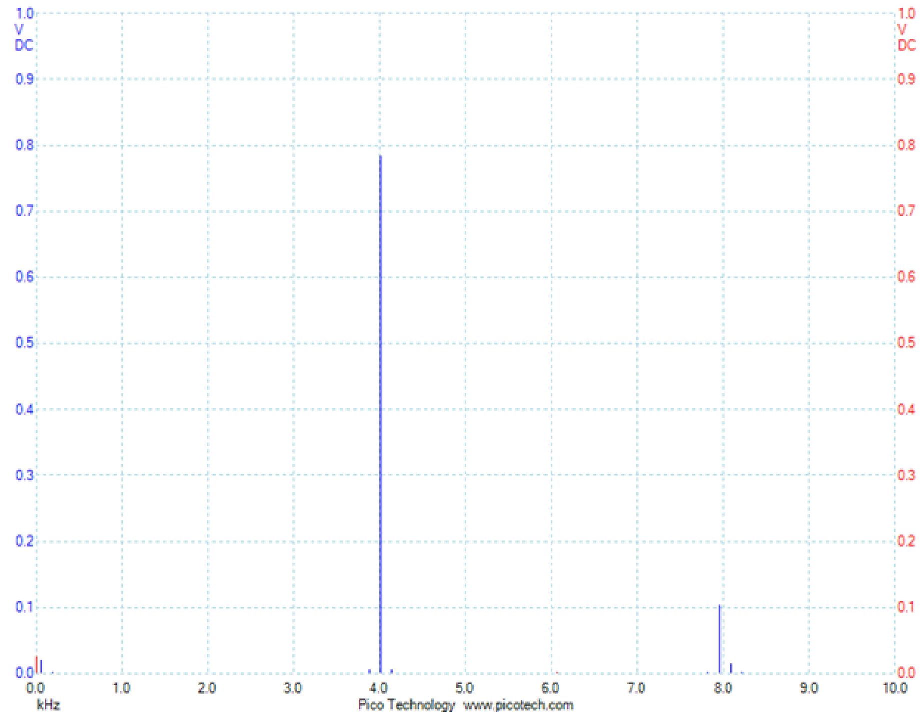
$$= a_1 \left[\cos 2\pi k + j \sin 2\pi k \right] + a_1 \left[\cos 2\pi k - j \sin 2\pi k \right]$$

$$= 2a_1 \cos 2\pi k, \quad k - 0,1,\ldots,N-1$$

a. The sum of two signals: one has positive frequency and the other one has negative frequency

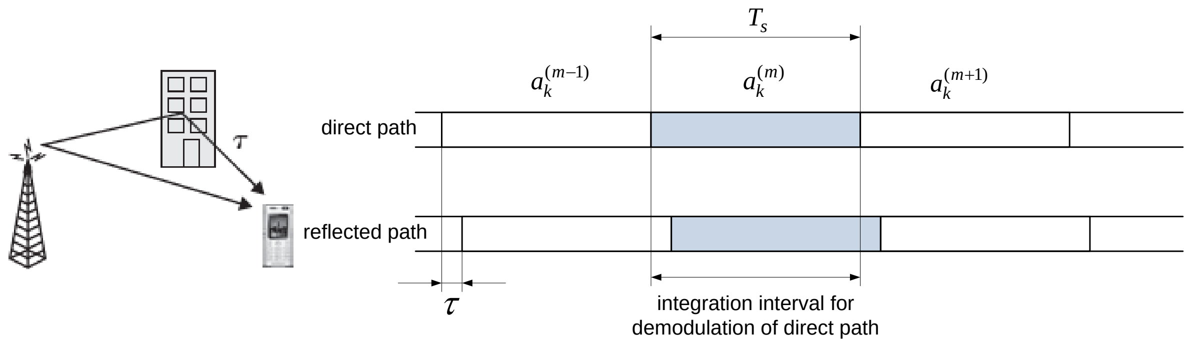




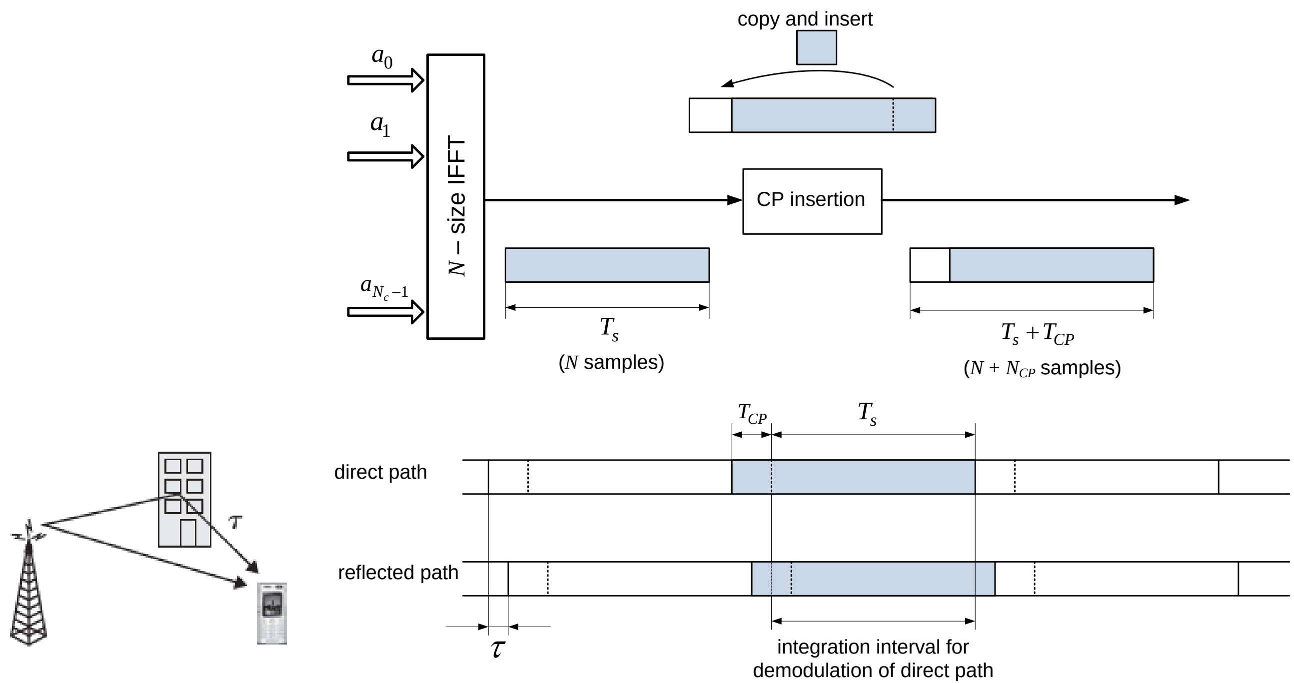


$$z = e^{j8\pi f t} + e^{-j8\pi f t}$$

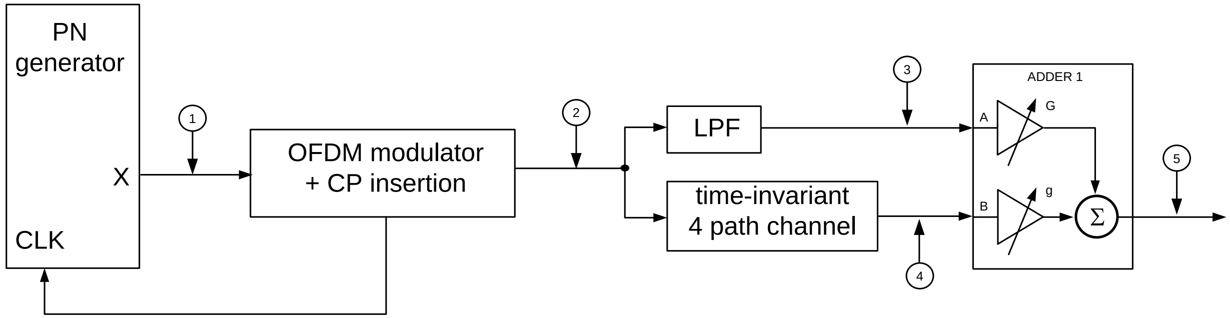
f = - 4 kHz + f = 4 kHz



a. time-dispersive channel
Source: Dahlman et. al, 3G Evolution HSPA and LTE for Mobile Broadband



b. Cyclic prefix insertion
Source: Dahlman et. al, 3G Evolution HSPA and LTE for Mobile Broadband



a. Block diagram

Sampling time: $T_{sample\_actual} = 7.8125[\mu s]$

clock signal period for PN generator: $T_{CLK\_PN} = 2 \cdot T_{sample\_actual} = 2 \times 7.8125 \times 10^{-6} = 15.625[\mu s]$

the duration of the bit at the output of the PN generator: $T_{CLK\_PN} = T_{b\_PN} = 15.625[\mu s]$

gross data rate at the output of the PN generator: $R_{b\_PN} = \frac{1}{T_{b\_PN}} = \frac{1}{15.625 \times 10^{-6}} = 64[kb/s]$

minimum spacing of subcarriers: $\Delta f = f_u = 1[kHz]$

duration of OFDM symbol without CP: $T_s = \frac{1}{\Delta f} = 1[ms]$

duration of OFDM symbol with CP: $T_{s\_CP} = 1.25 \cdot T_s = 1.25[ms]$

number of subcarriers: $N_c = 20$ modulation: 4QAM the active part of the frame lasts : $T_{burst} = 40 \cdot T_{b\_PN} = 40 \times 15.625[\mu s] = 625.0[\mu s]$

frame time: $T_f = 80 \cdot T_{b\_PN} = 80 \times 15.625[\mu s] = 1250[\mu s]$

the average bit rate transmitted over the *i*-th subcarrier: $R_{b\_CP\_i\_net} = \frac{k}{T_{s\_CP}} = \frac{2}{1.25 \times 10^{-3}} = 1.6[kb/s]$

the total bit rate transmitted during 1 OFDM symbol:

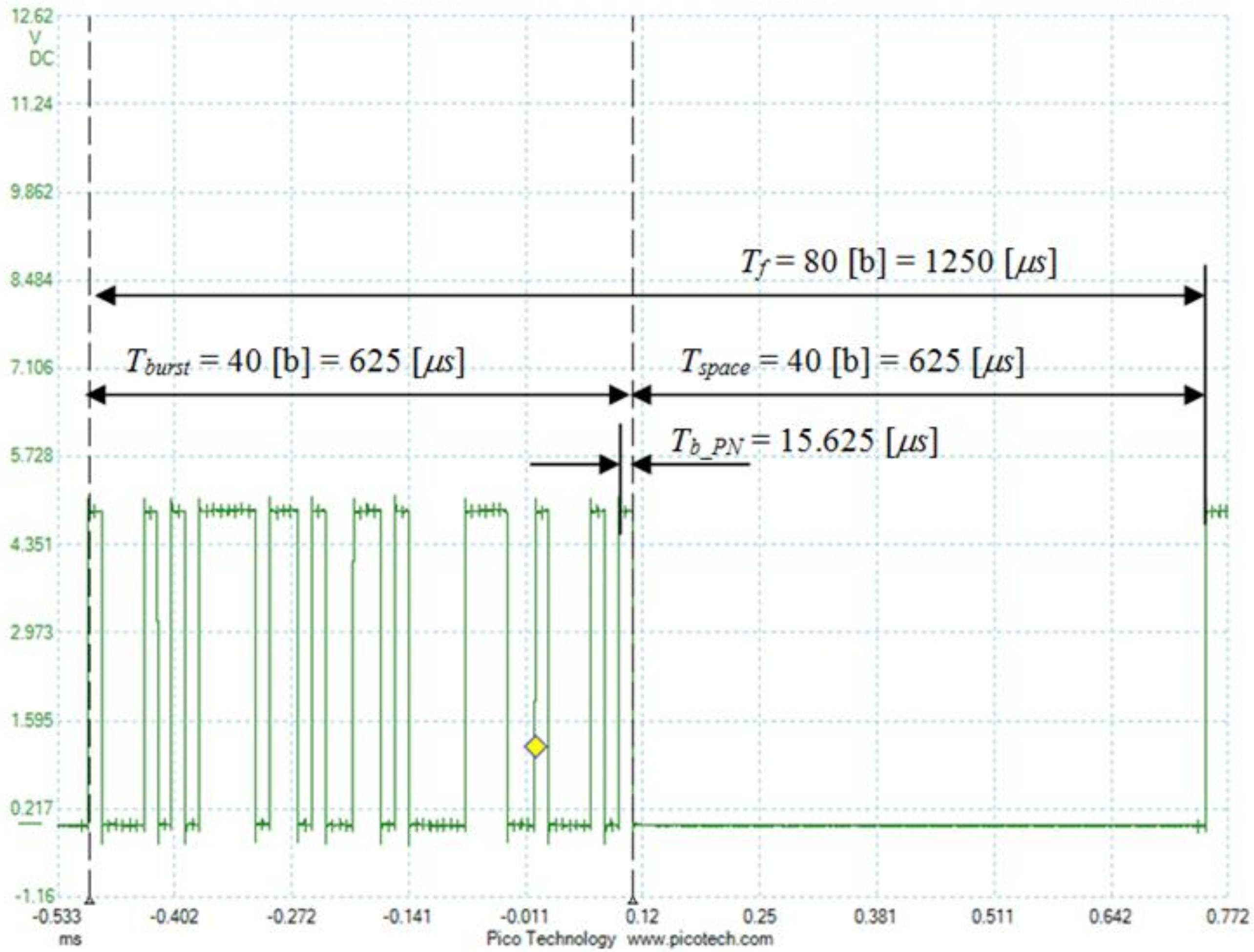
$$R_{b\_CP\_net} = N_c \cdot R_{b\_CP\_i\_net} = 20 \times 1.6[kb/s] = 32[kb/s]$$

total bit rate without CP: $R_{b\_net} = N_c \cdot R_{b\_\_i\_net} = N_c \cdot \frac{k}{T_s} = 20 \times \frac{2}{10^{-3}} = 40[kb/s]$

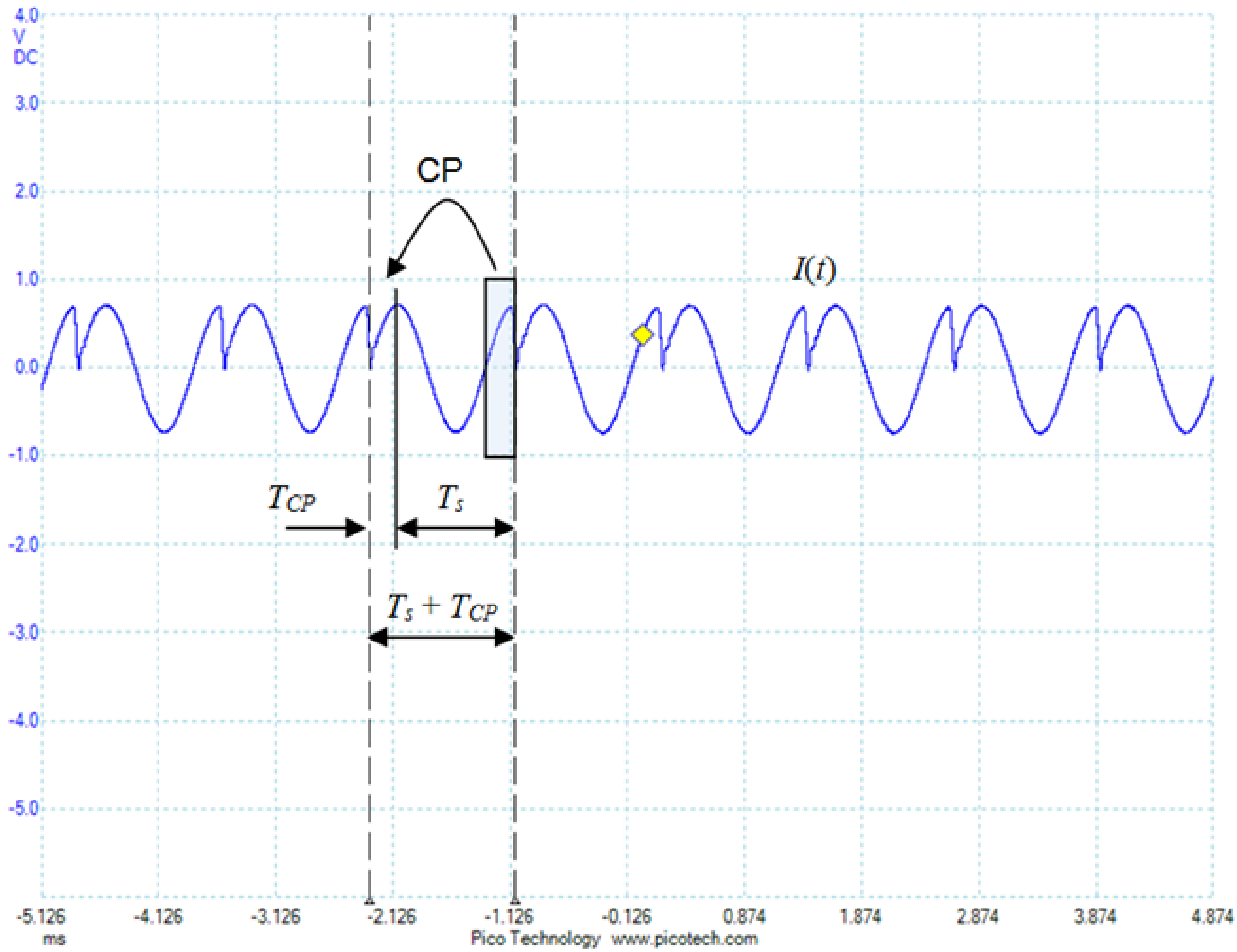
$$\frac{R_{b\_net}}{R_{b\_CP\_net}} = \frac{40[kb/s]}{32[kb/s]} = 1.25$$

b. PAPR approximate value: $PAPR_{measured} = \frac{A_{peak}^2}{A_{avg}^2}$

the exact value of the maximum PAPR for the square M-QAM: $PAPR_{\max(QAM-OFDM)} = \frac{3N_c\left(\sqrt{M}-1\right)}{\sqrt{M}+1}$

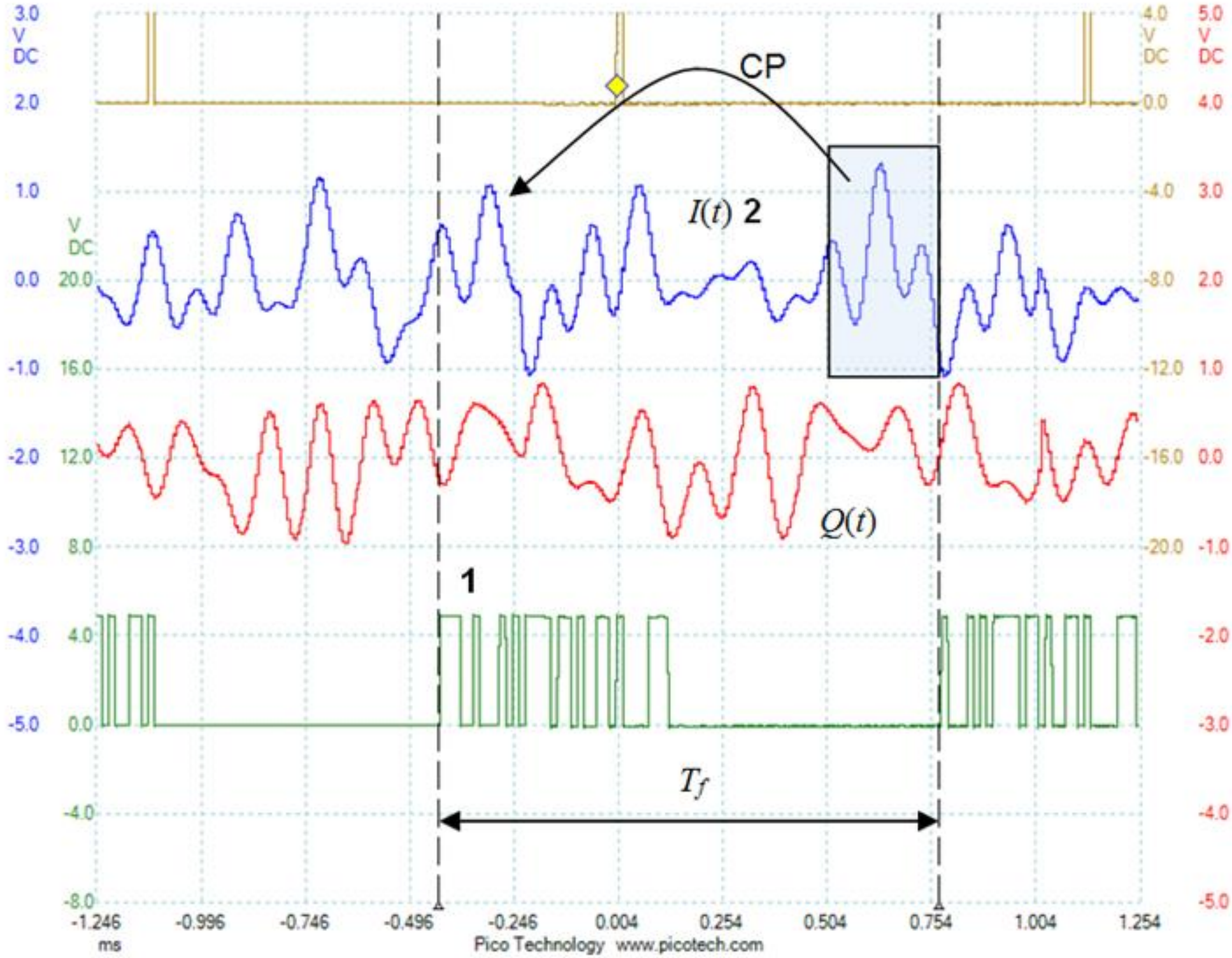

12.62
V
DC
11.24
9.862
8.484
7.106
5.728
4.351
2.973
1.595
0.217
-1.16
-0.533
ms
-0.402
-0.272
-0.141
-0.011
0.12
0.25
0.381
0.511
0.642
0.772
Pico Technology www.picotech.com
T_f = 80 [b] = 1250 [μs]
T_{burst} = 40 [b] = 625 [μs]
T_{space} = 40 [b] = 625 [μs]
T_{b_PN} = 15.625 [μs]

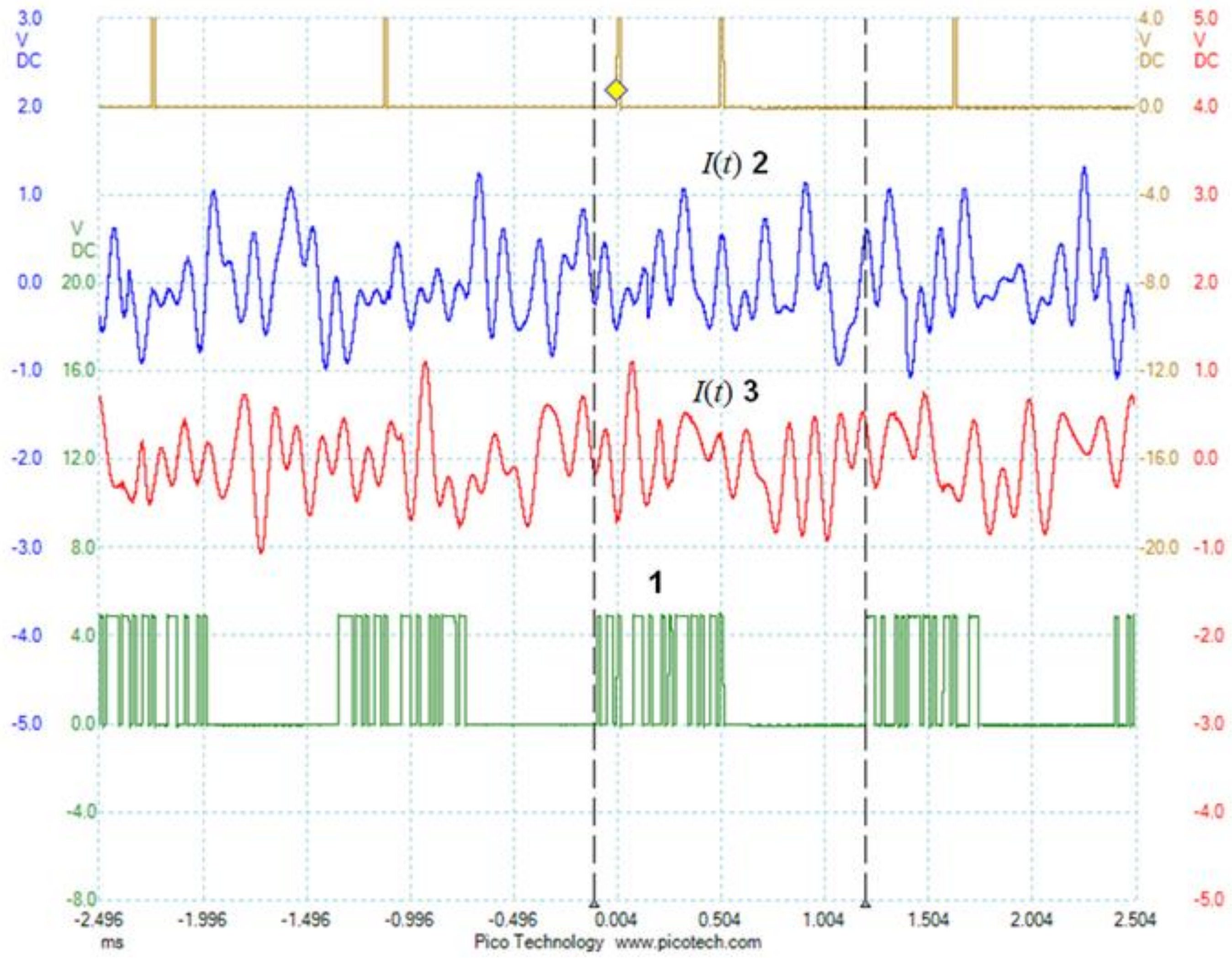
data frame



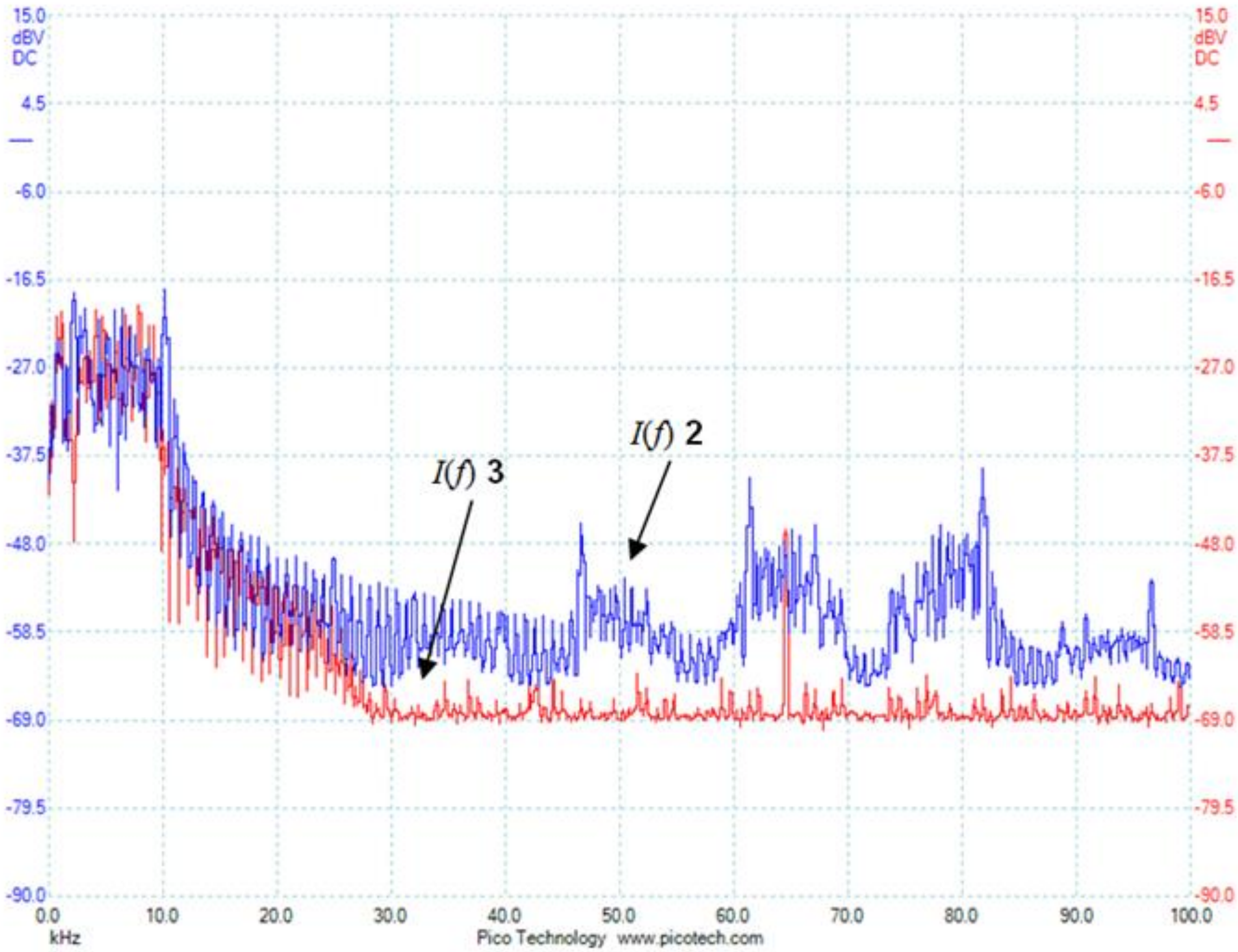
IDFT output for TESTPATTERN[1] = 100 + j0

f = 1 kHz

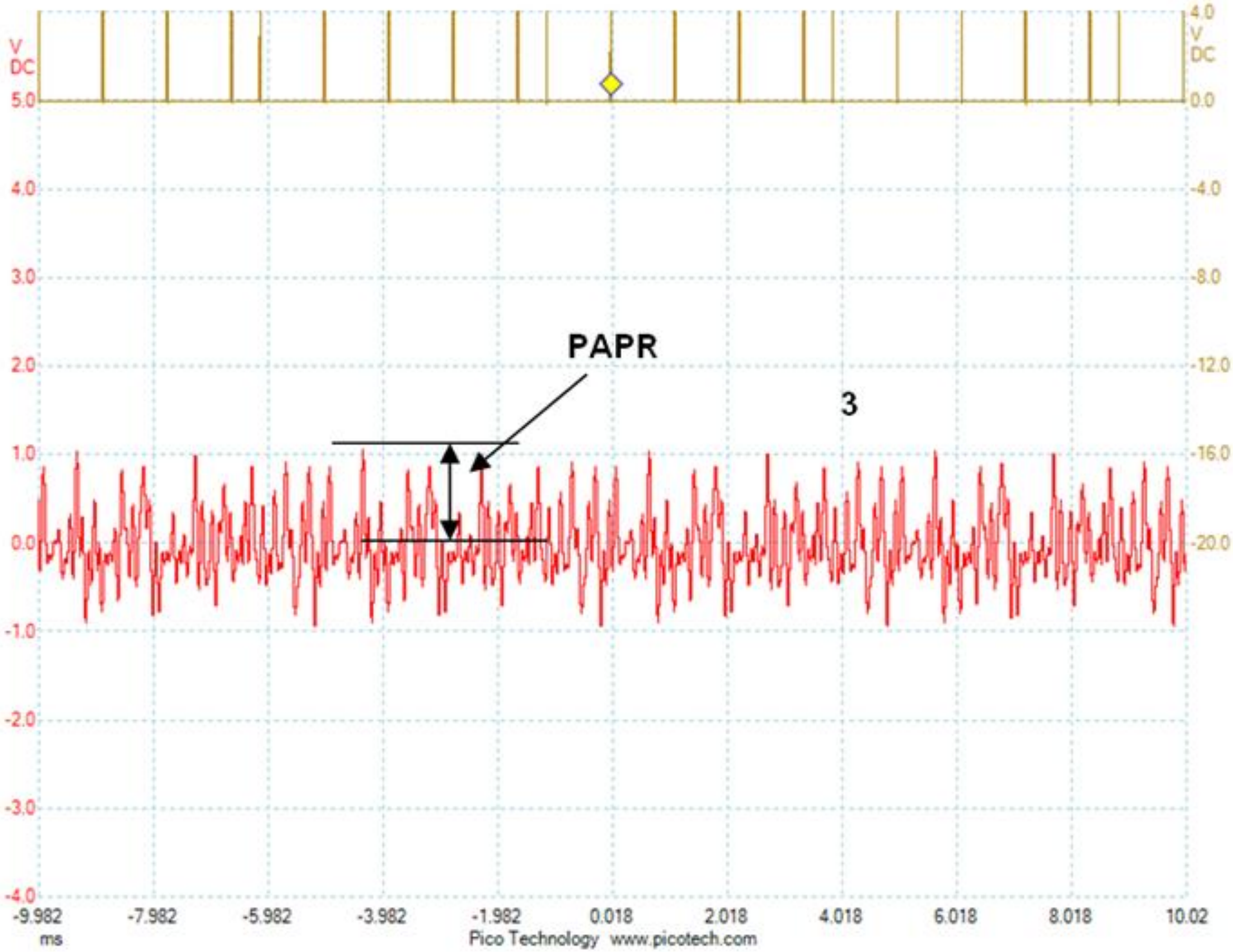

CP
I(t) 2
Q(t)
1
T_f
Pico Technology www.picotech.com
ms
-1.246
-0.996
-0.746
-0.496
-0.246
0.004
0.254
0.504
0.754
1.004
1.254



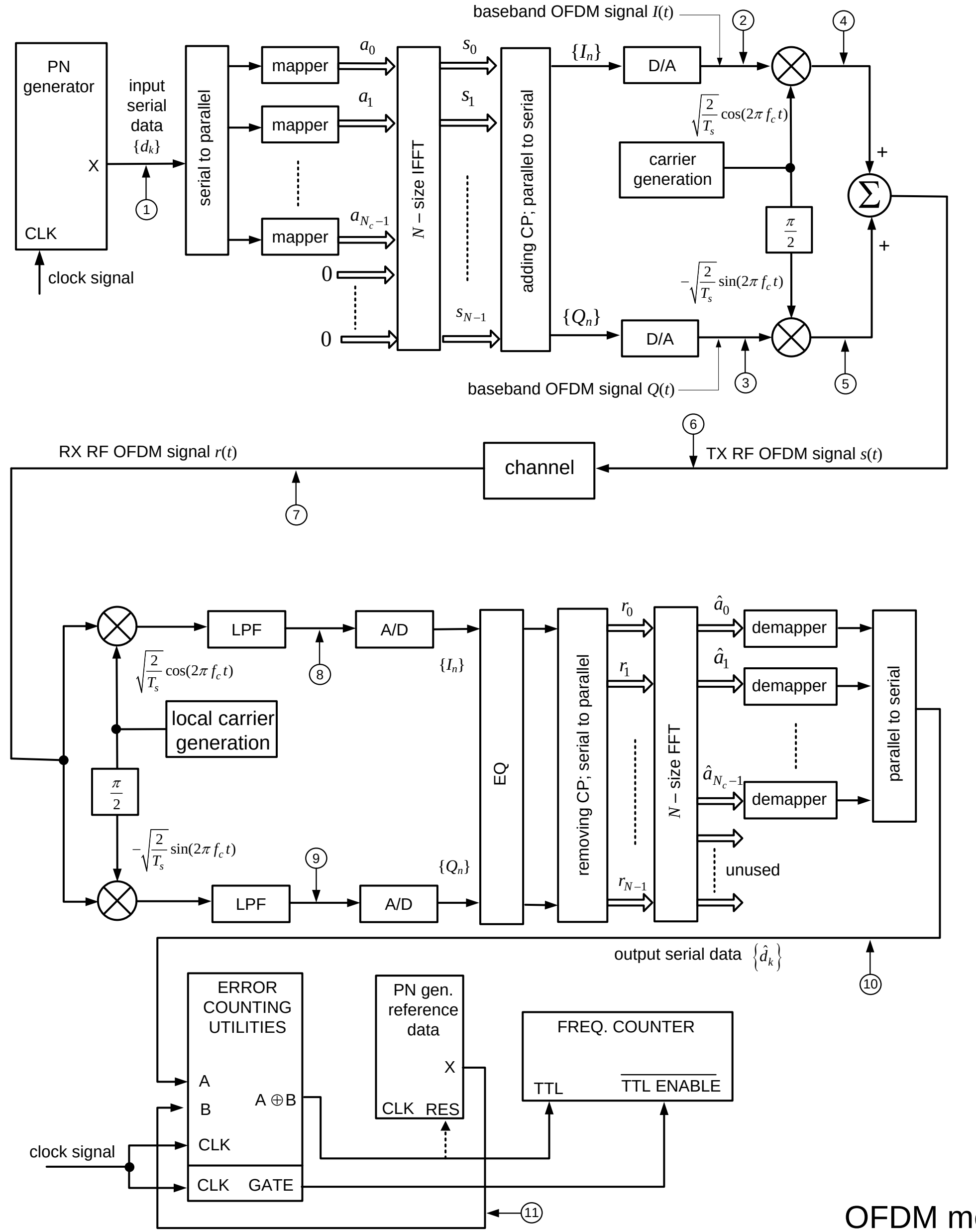
OFDM signal in baseband



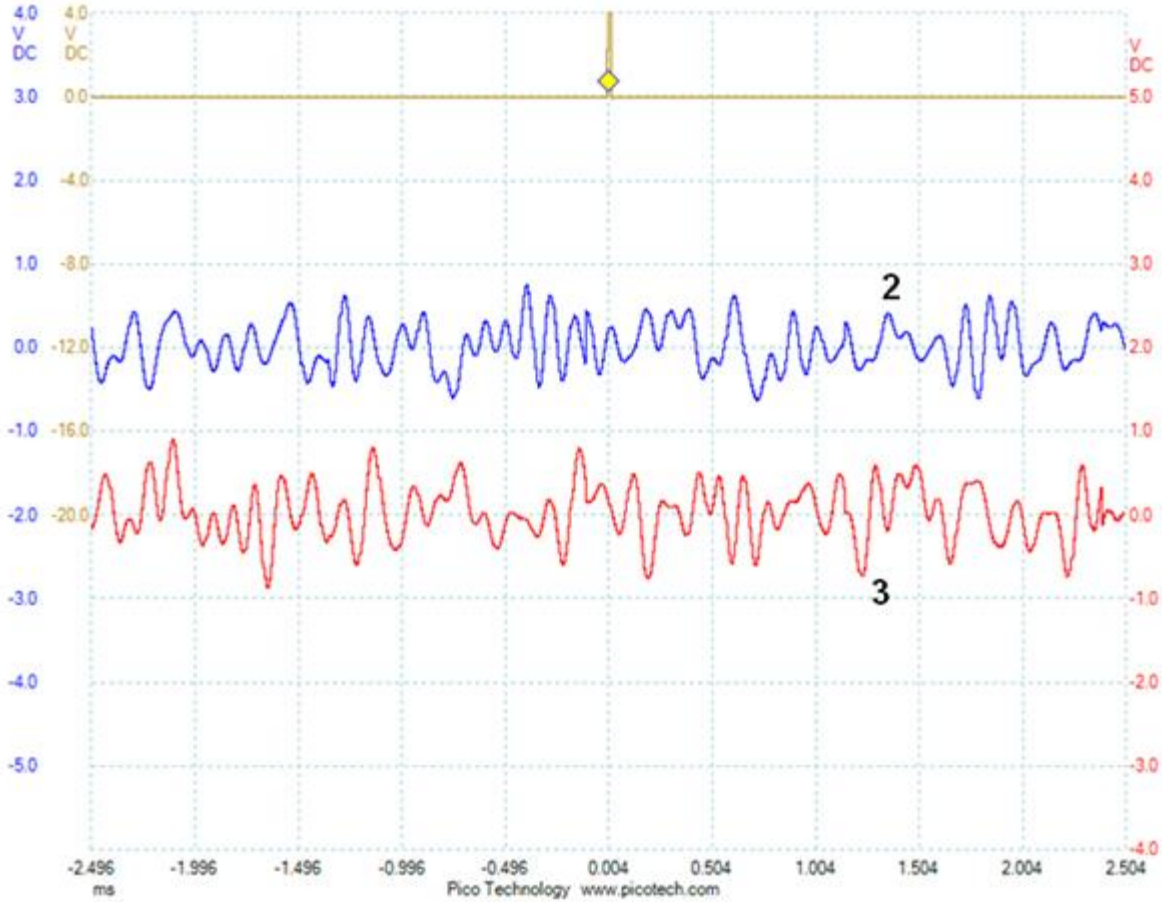
OFDM baseband spektrum



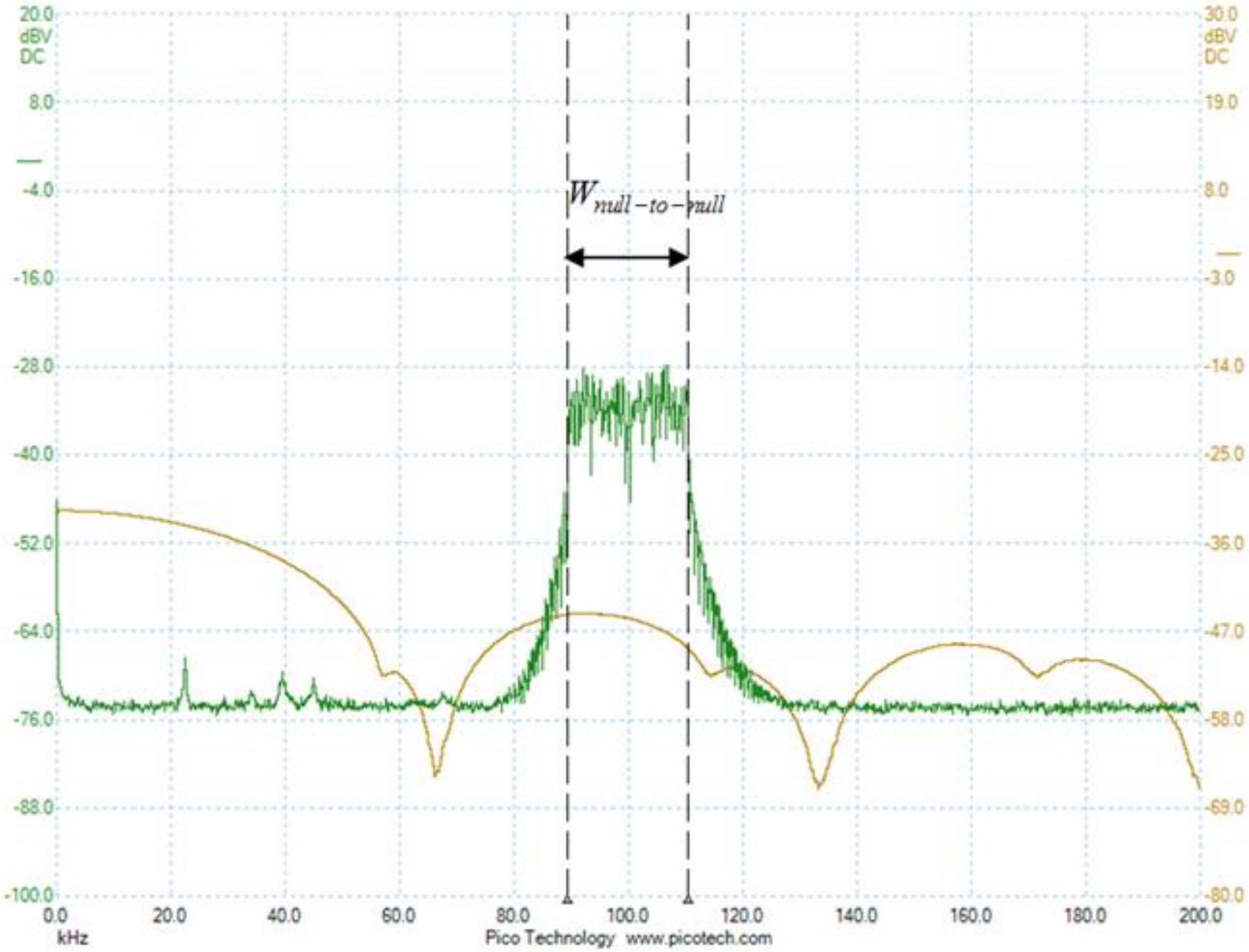
output OFDM signal in baseband


PN generator
X
CLK
clock signal
input serial data $\{d_k\}$
serial to parallel
mapper
mapper
mapper
a_0
a_1
a_{N_c-1}
0
0
N – size IFFT
s_0
s_1
s_{N-1}
adding CP; parallel to serial
$\{I_n\}$
$\{Q_n\}$
D/A
D/A
baseband OFDM signal $I(t)$
baseband OFDM signal $Q(t)$
$\sqrt{\frac{2}{T_s}}\cos(2\pi f_c t)$
carrier generation
$\frac{\pi}{2}$
$-\sqrt{\frac{2}{T_s}}\sin(2\pi f_c t)$
TX RF OFDM signal $s(t)$
channel
RX RF OFDM signal $r(t)$
LPF
LPF
A/D
A/D
local carrier generation
EQ
removing CP; serial to parallel
r_0
r_1
r_{N-1}
N – size FFT
$\hat{a}_0$
$\hat{a}_1$
$\hat{a}_{N_c-1}$
demapper
demapper
demapper
unused
parallel to serial
output serial data $\{\hat{d}_k\}$
ERROR COUNTING UTILITIES
A
B
CLK
A ⊕B
CLK GATE
PN gen. reference data
X
CLK RES
FREQ. COUNTER
TTL
TTL ENABLE

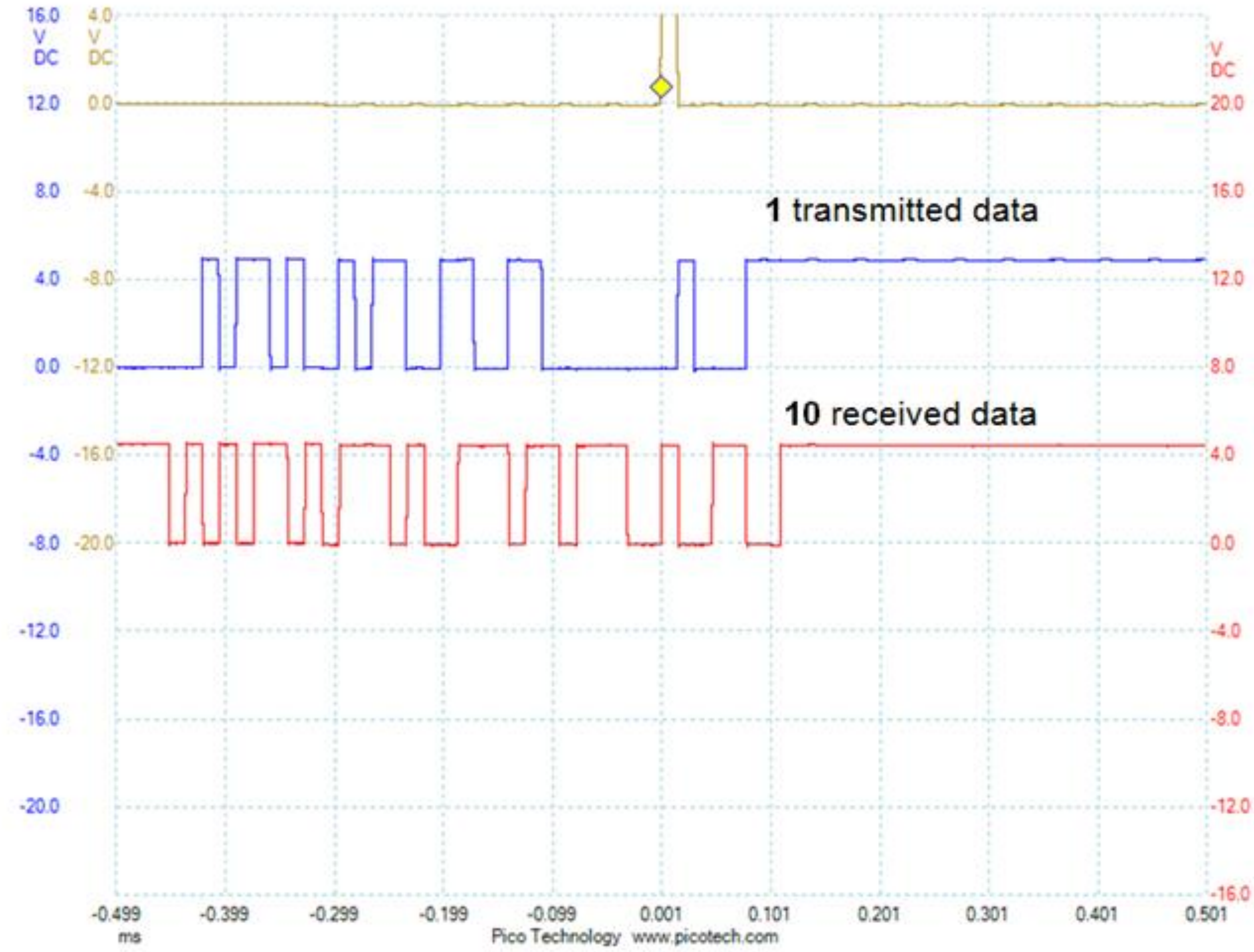
OFDM modem



output OFDM signal in bandpass



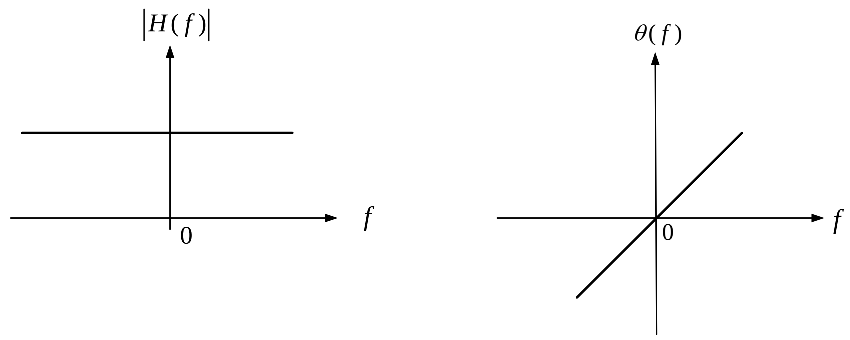
Bandpass OFDM spektrum



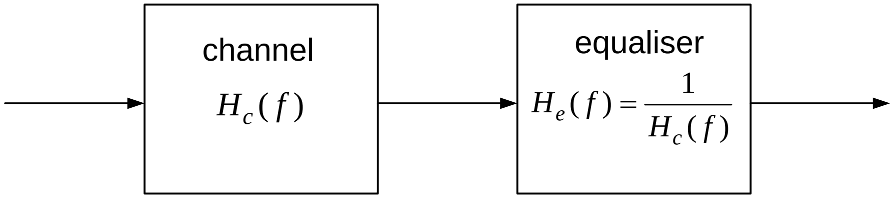
transmitted and received data without EQ

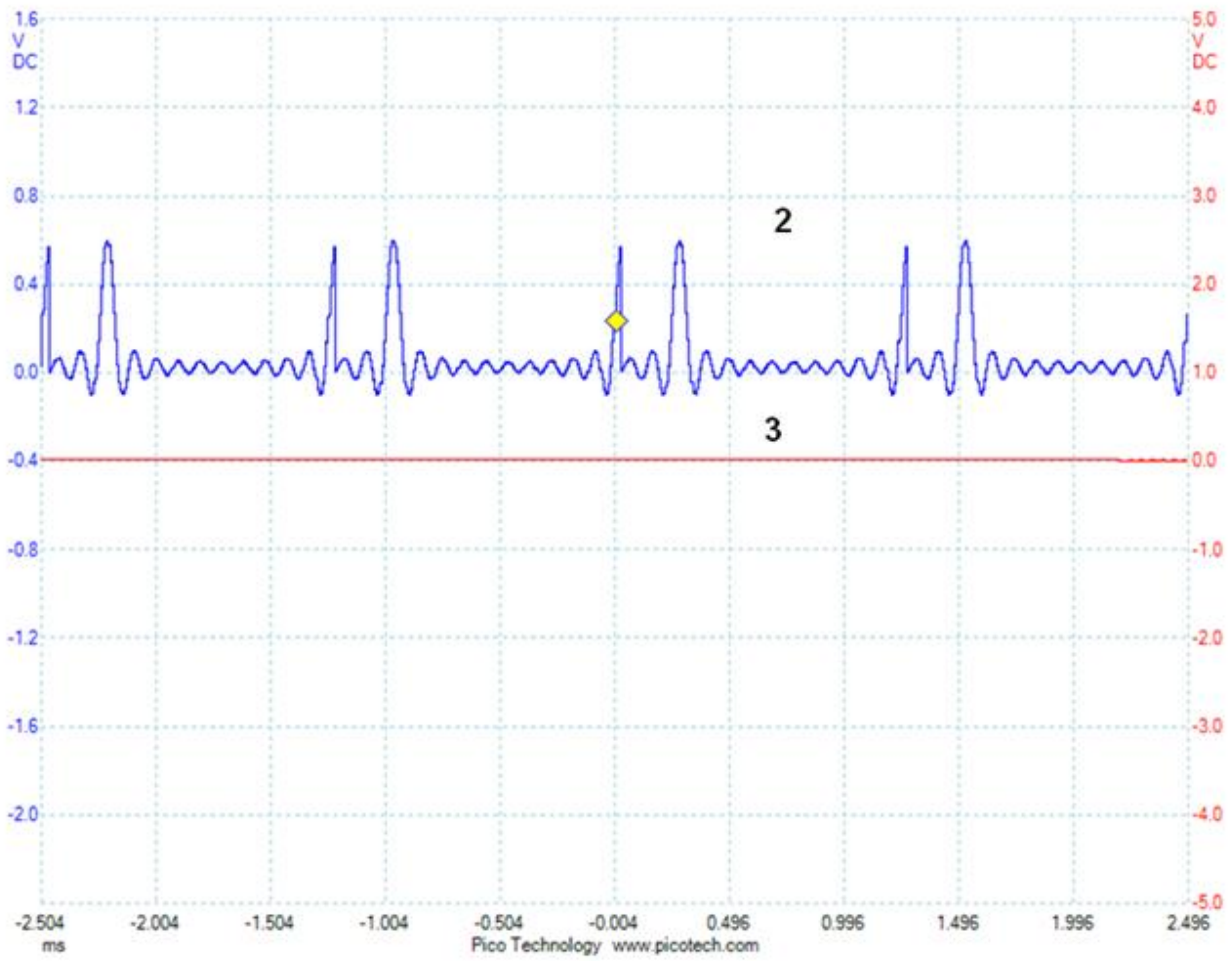
a. OFDM signal bandwidth: $W_{null-to-null} = \frac{N_c + 1}{T_s} = \frac{20+1}{10^{-3}} = 21[kHz]$

b. condition of undistorted transmission: $Y(f) = K\,X(f)e^{-j2\pi f t_0} \Rightarrow H(f) = \frac{Y(f)}{X(f)} = K\,e^{-j2\pi f t_0} = |H(f)|e^{-j\theta(f)}$

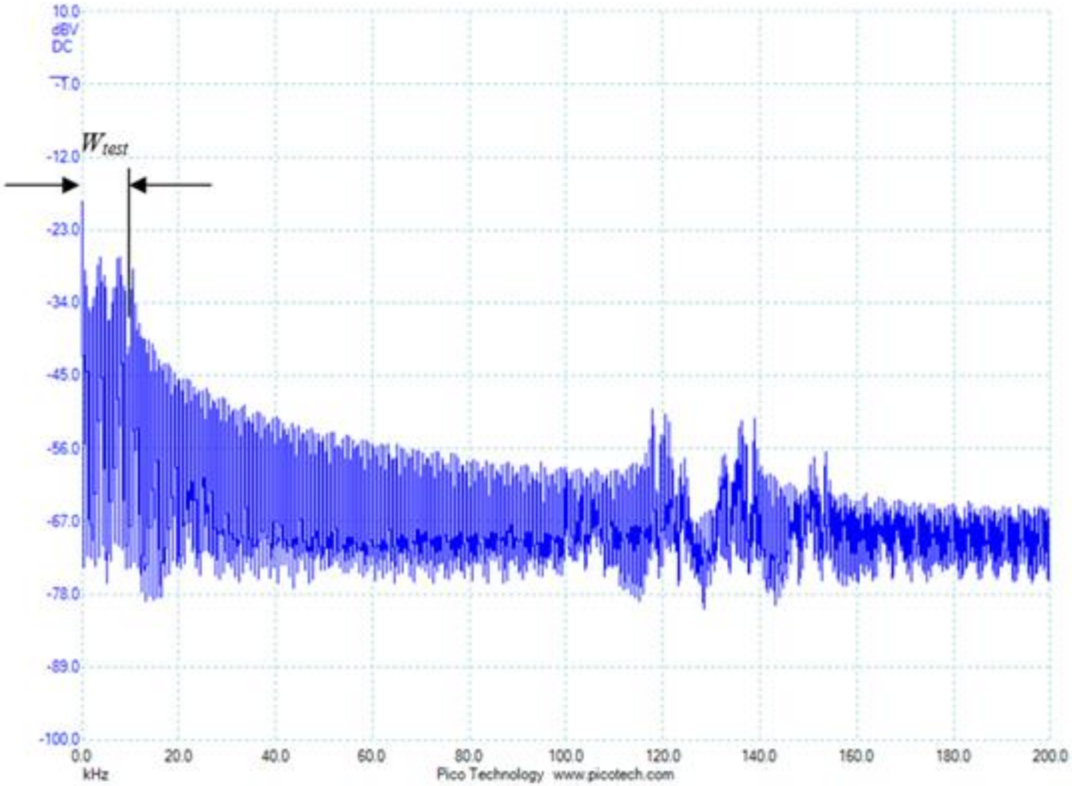


c. equalization strategy:

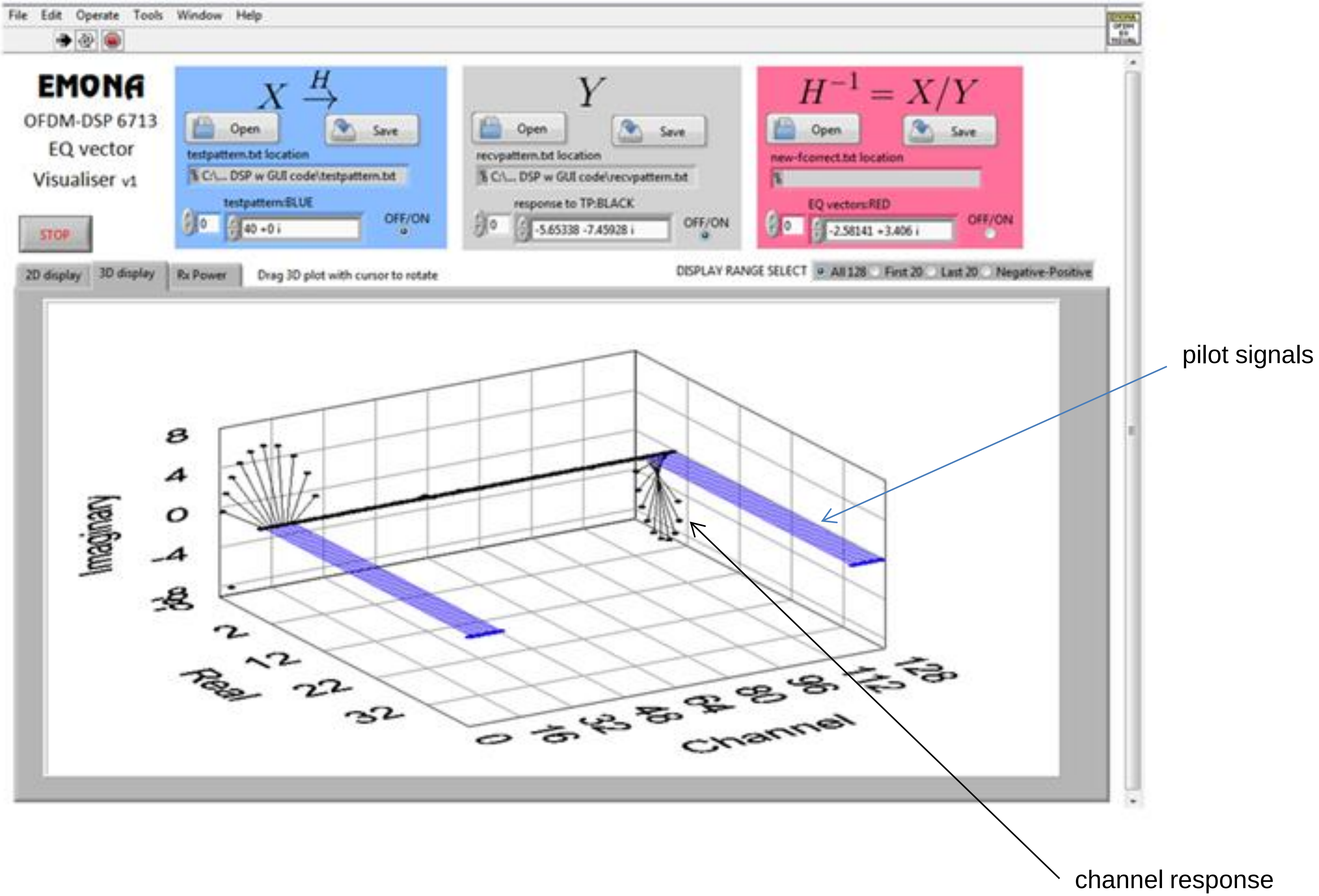


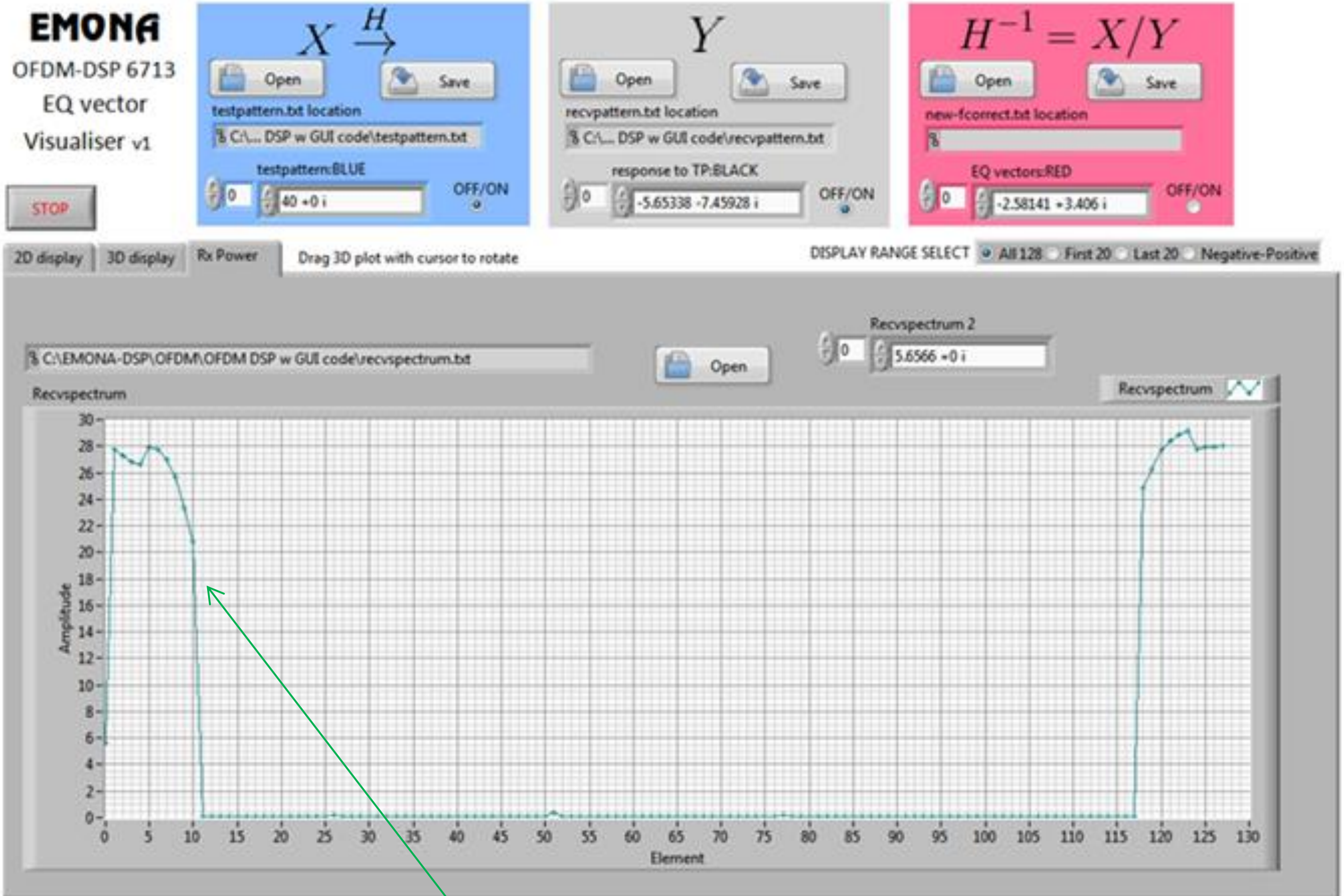


pilot signals in the time domain

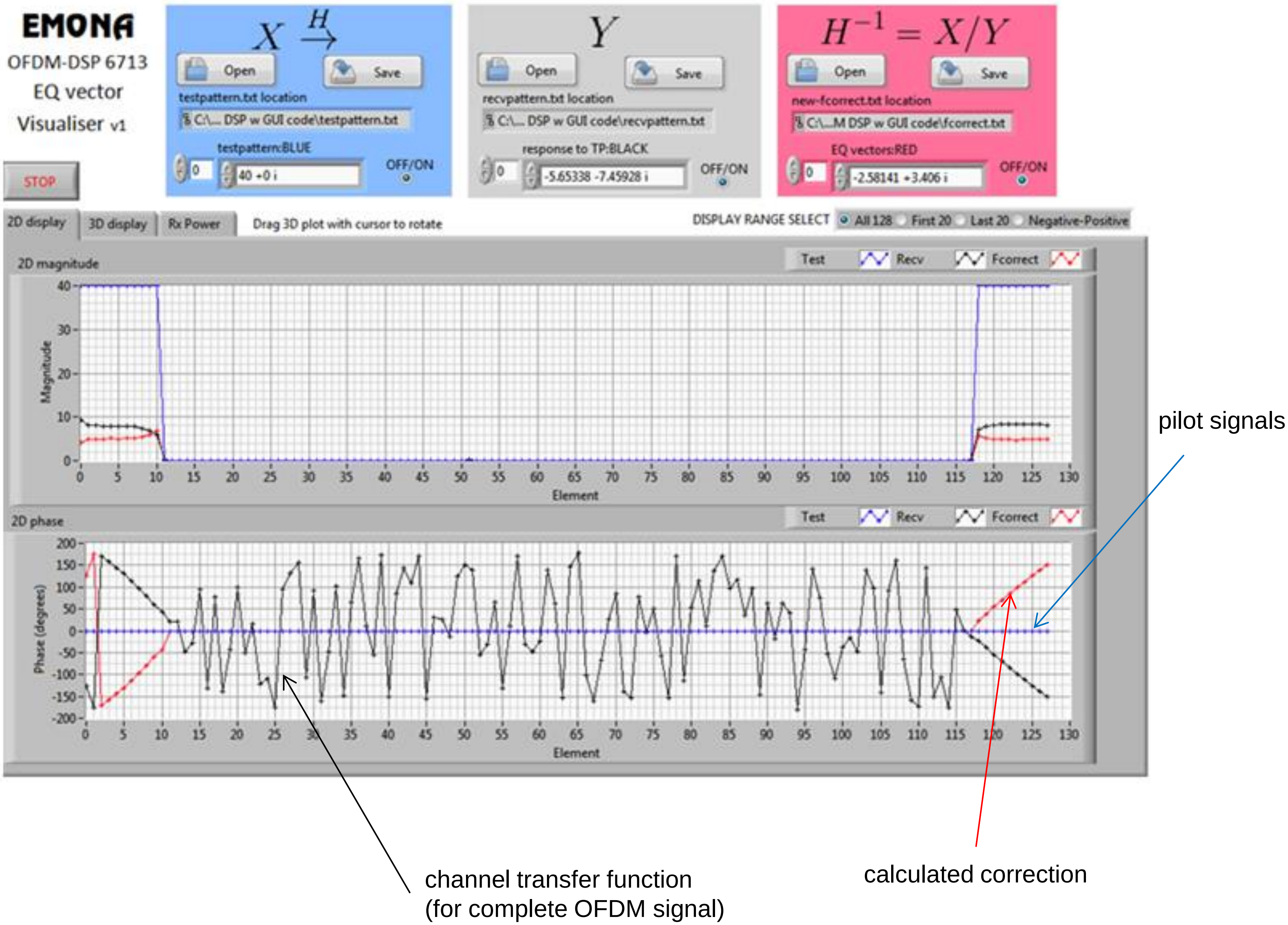


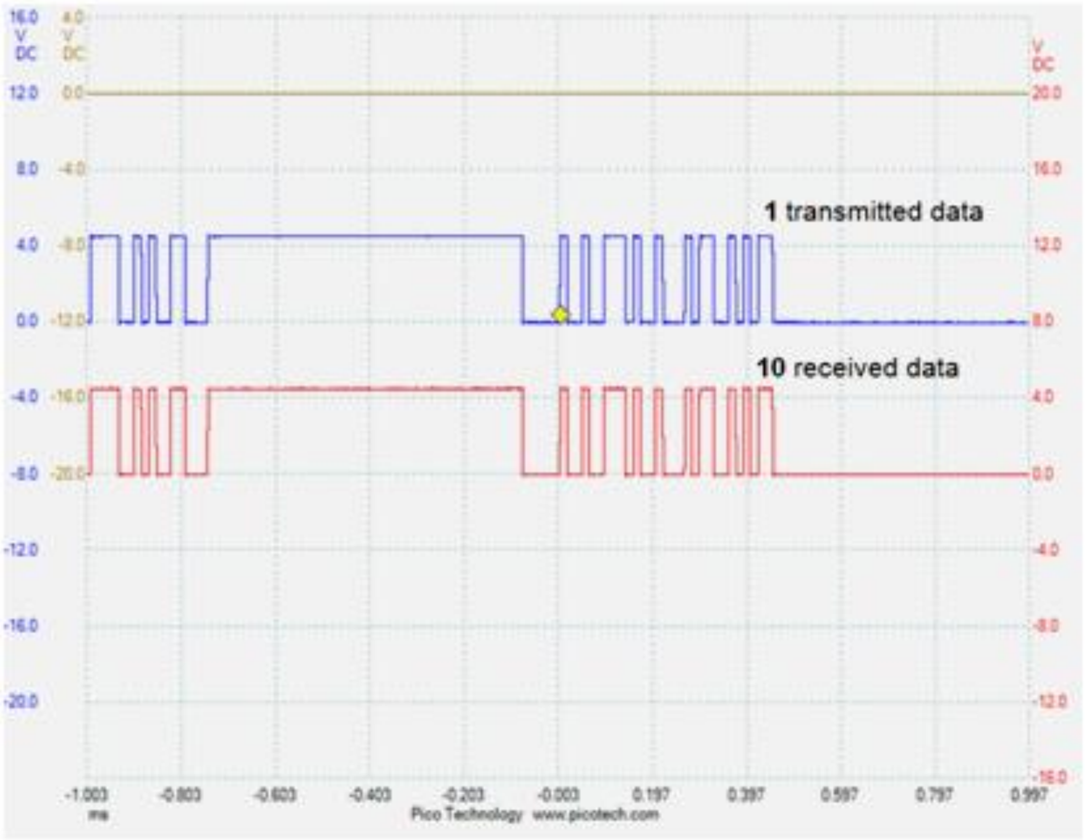
pilot signals in the frequency domain


File Edit Operate Tools Window Help
EMONA
OFDM-DSP 6713
EQ vector
Visualiser v1
STOP
$X \xrightarrow{H}$
Open
Save
testpattern.txt location
C:\... DSP w GUI code\testpattern.txt
testpattern:BLUE
40 +0 i
OFF/ON
Y
recvpattern.txt location
C:\... DSP w GUI code\recvpattern.txt
response to TP:BLACK
-5.65338 -7.45928 i
$H^{-1} = X/Y$
new-fcorrect.txt location
EQ vectors:RED
-2.58141 +3.406 i
2D display
3D display
Rx Power
Drag 3D plot with cursor to rotate
DISPLAY RANGE SELECT
All 128
First 20
Last 20
Negative-Positive
Imaginary
Real
Channel
pilot signals
channel response

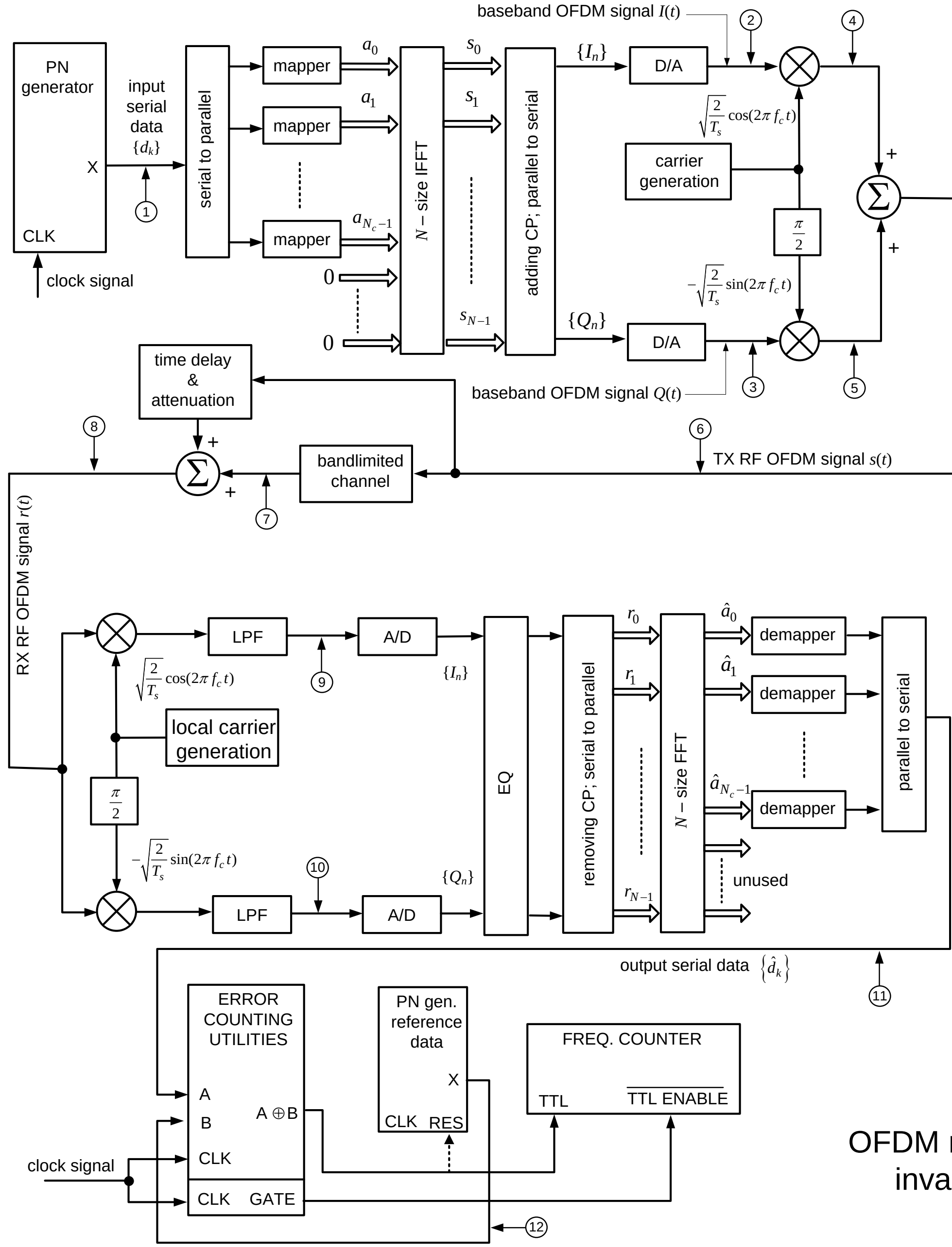


channel transfer function (only for pilot signals)


EMONA
OFDM-DSP 6713
EQ vector
Visualiser v1
STOP
$X \xrightarrow{H}$
Open
Save
testpattern.txt location
C:\... DSP w GUI code\testpattern.txt
testpattern:BLUE
40 +0 i
OFF/ON
Y
recvpattern.txt location
C:\... DSP w GUI code\recvpattern.txt
response to TP:BLACK
-5.65338 -7.45928 i
$H^{-1} = X/Y$
new-fcorrect.txt location
C:\...M DSP w GUI code\fcorrect.txt
EQ vectors:RED
-2.58141 +3.406 i
2D display
3D display
Rx Power
Drag 3D plot with cursor to rotate
DISPLAY RANGE SELECT
All 128
First 20
Last 20
Negative-Positive
2D magnitude
Test
Recv
Fcorrect
Magnitude
Element
2D phase
Phase (degrees)
pilot signals
channel transfer function
(for complete OFDM signal)
calculated correction



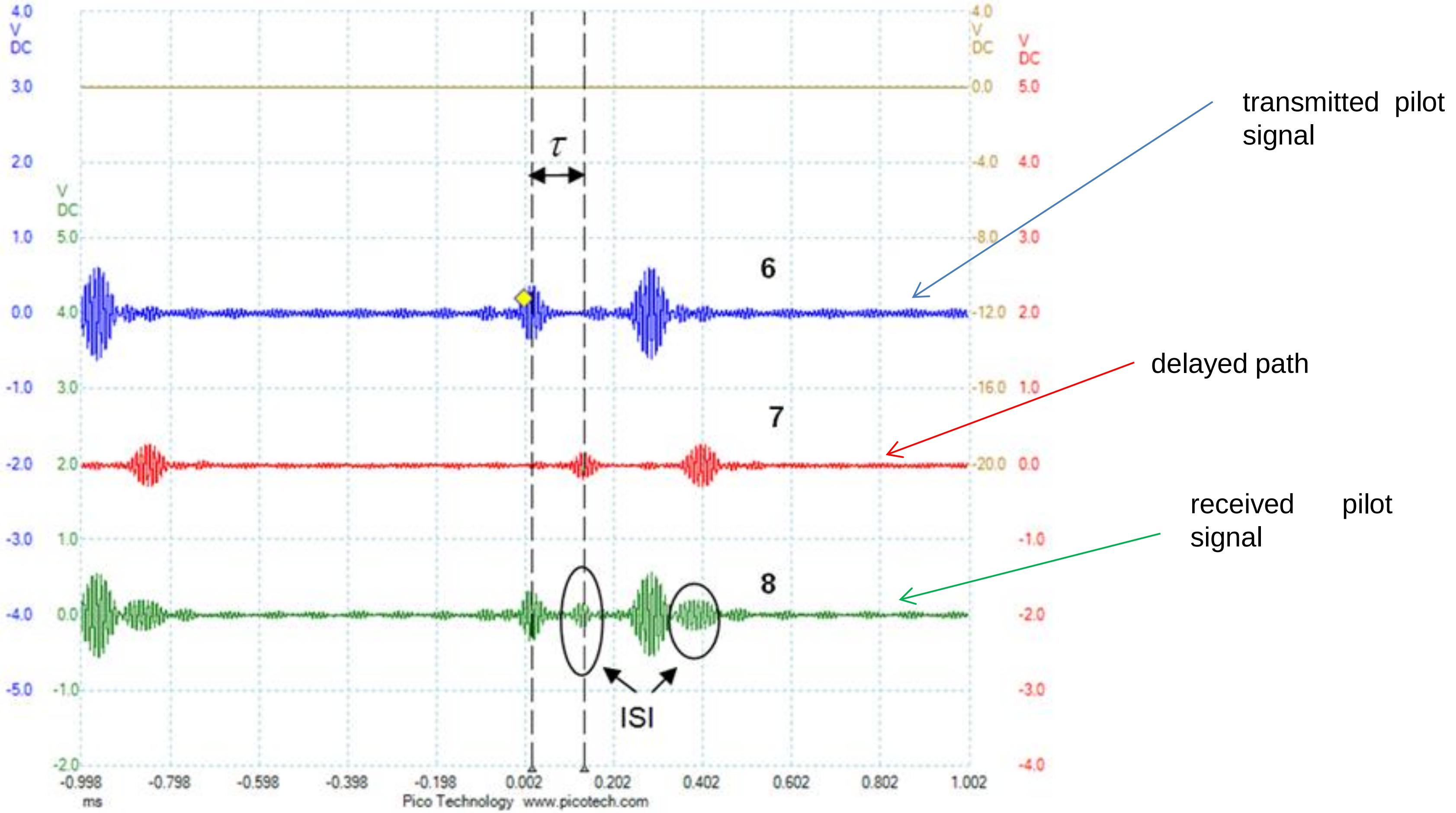
transmitted and received data with EQ

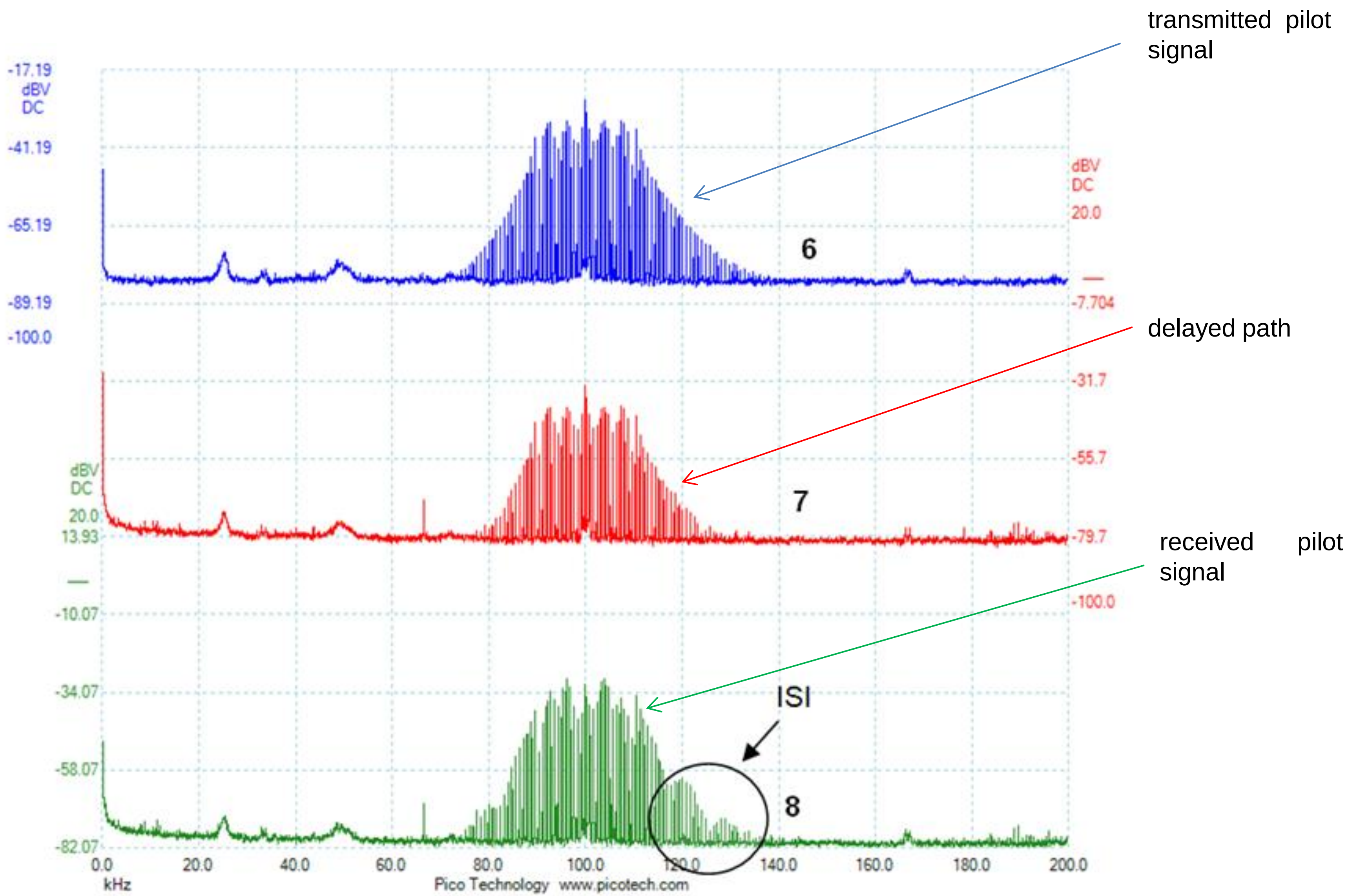


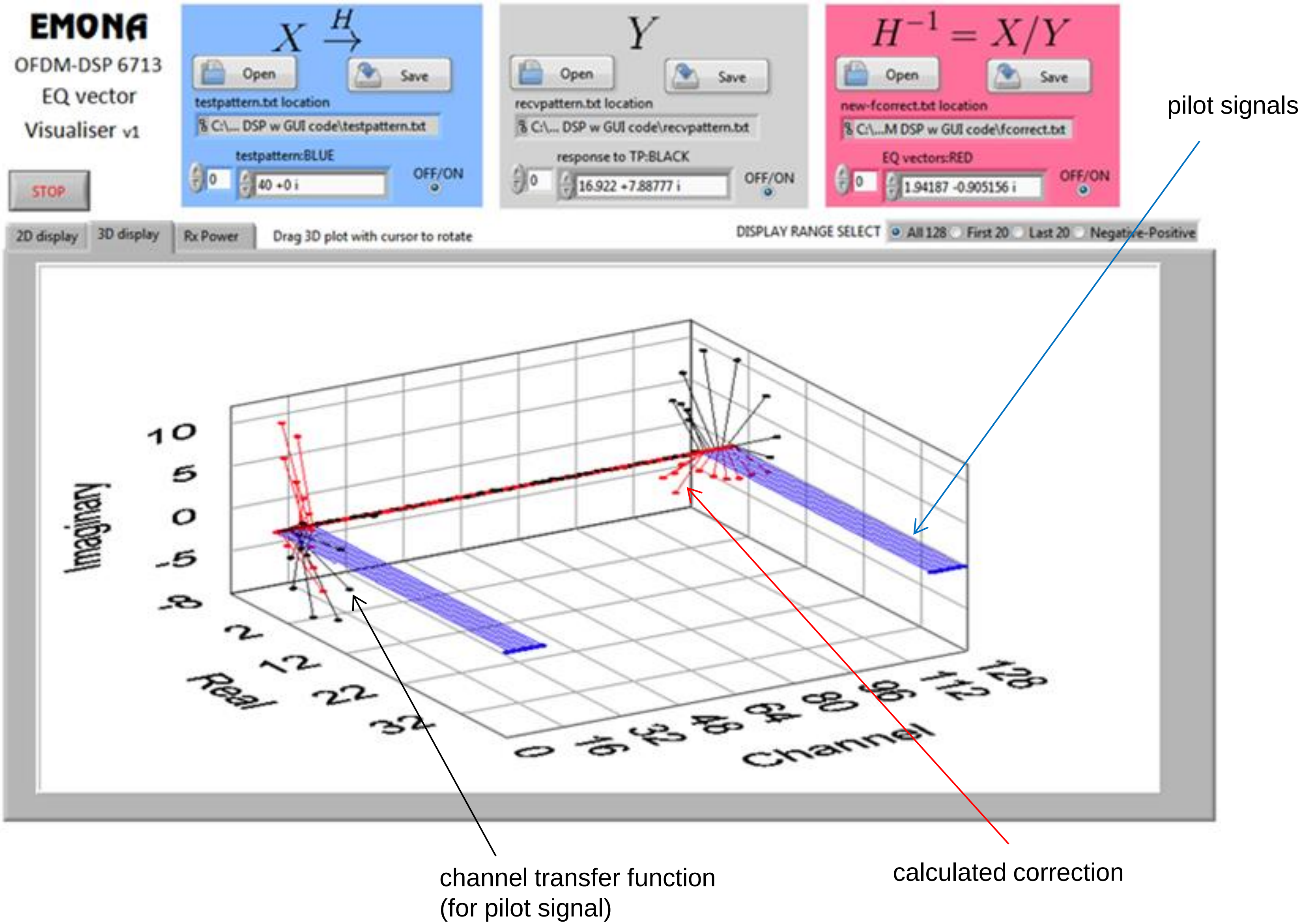
OFDM modem and time-invariant channel

path #	1	2	3	4
delay [μs]	T1	T2	T3	T4
	100	0	0	0
attenuation [dB]	A1	A2	A3	A4
	6	OFF	OFF	OFF

one delayed path $\tau < T_{CP}$



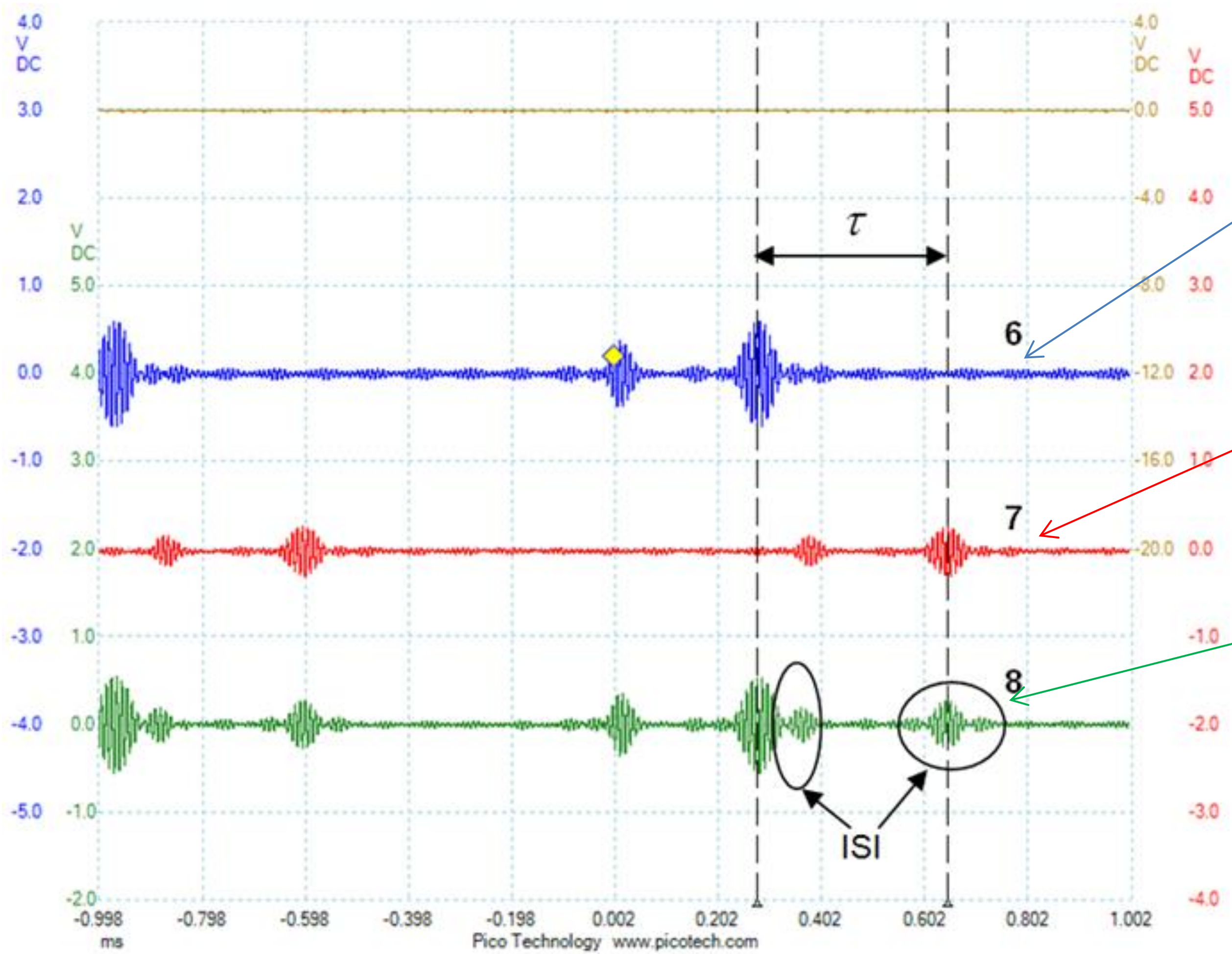

transmitted pilot signal
delayed path
received pilot signal
ISI
6
7
8
-17.19 dBV DC
-41.19
-65.19
-89.19
-100.0
dBV DC
20.0
-7.704
-31.7
-55.7
-79.7
-100.0
dBV DC
20.0
13.93
-10.07
-34.07
-58.07
-82.07
0.0
20.0
40.0
60.0
80.0
100.0
120.0
140.0
160.0
180.0
200.0
kHz
Pico Technology www.picotech.com



EMONA
OFDM-DSP 6713
EQ vector
Visualiser v1
STOP
$X \xrightarrow{H}$
Open
Save
testpattern.txt location
C:\... DSP w GUI code\testpattern.txt
testpattern:BLUE
40 +0 i
OFF/ON
Y
recvpattern.txt location
C:\... DSP w GUI code\recvpattern.txt
response to TP:BLACK
16.922 +7.88777 i
$H^{-1} = X/Y$
new-fcorrect.txt location
C:\...M DSP w GUI code\fcorrect.txt
EQ vectors:RED
1.94187 -0.905156 i
2D display
3D display
Rx Power
Drag 3D plot with cursor to rotate
DISPLAY RANGE SELECT
All 128
First 20
Last 20
Negative-Positive
Imaginary
Real
Channel
pilot signals
channel transfer function
(for pilot signal)
calculated correction

path #	1	2	3	4
delay [μs]	T1	T2	T3	T4
	350	0	0	0
attenuation [dB]	A1	A2	A3	A4
	6	OFF	OFF	OFF

one delayed path $\tau > T_{CP}$



channel transfer function (only for pilot signals)