Fourierov rad

$$x(\lambda) = \tfrac{1}{2}a_0 + a_1\cos\lambda + a_2\cos 2\lambda + a_3\cos 3\lambda + \cdots + b_1\sin\lambda + b_2\sin 2\lambda + b_3\sin 3\lambda + \cdots \quad (A.1)$$

exponenciálny tvar:

$$x(t) = \frac{a_0}{2} + \frac{1}{2}\sum_{n=1}^{\infty}\left[(a_n - jb_n)e^{j2\pi nf_0 t} + (a_n + jb_n)e^{-j2\pi nf_0 t}\right] \quad (A.11)$$

komplexné koeficienty:

$$c_n = \begin{cases} \tfrac{1}{2}(a_n - jb_n) & \text{for } n > 0 \\ \dfrac{a_0}{2} & \text{for } n = 0 \\ \tfrac{1}{2}(a_n + jb_n) & \text{for } n < 0 \end{cases} \quad (A.12)$$

$$c_n = |c_n|e^{j\theta_n} \quad (A.17)$$

$$c_{-n} = |c_n|e^{-j\theta_n} \quad (A.18)$$

potom:

$$x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j2\pi nf_0 t} \quad (A.13)$$

$$c_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-j2\pi nf_0 t}\,dt \quad (A.14)$$

nakoľko:

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} e^{j(n-m)2\pi f_0 t}\,dt = \delta_{nm} = \begin{cases} 1 & \text{for } n = m \\ 0 & \text{for } n \neq m \end{cases} \quad (A.15)$$

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)e^{-j2\pi mf_0 t}\,dt = \sum_{n=-\infty}^{\infty} c_n\delta_{nm} = c_m \quad (A.16)$$

Fourierova transformácia

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt \quad (A.26)$$

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,df \quad (A.27)$$

$x(t)$	$X(f)$
1. $\delta(t)$	1
2. 1	$\delta(f)$
3. $\cos 2\pi f_0 t$	$\frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$
4. $\sin 2\pi f_0 t$	$\frac{1}{2j}[\delta(f - f_0) - \delta(f + f_0)]$
5. $\delta(t - t_0)$	$\exp(-j2\pi ft_0)$
6. $\exp(j2\pi f_0 t)$	$\delta(f - f_0)$
7. $\exp(-a\lvert t\rvert),\quad a > 0$	$\dfrac{2a}{a^2 + (2\pi f)^2}$
8. $\exp\left[-\pi\left(\frac{t}{T}\right)^2\right]$	$T\exp[-\pi(fT)^2]$
9. $u(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases}$	$\frac{1}{2}\delta(f) + \dfrac{1}{j2\pi f}$
10. $\exp(-at)\,u(t),\quad a > 0$	$\dfrac{1}{a + j2\pi f}$
11. $t\exp(-at)\,u(t),\quad a > 0$	$\dfrac{1}{(a + j2\pi f)^2}$
12. $\text{rect}\left(\frac{t}{T}\right)$	$T\,\text{sinc}\,fT$
13. $\cos 2\pi f_0 t\left[\text{rect}\left(\frac{t}{T}\right)\right]$	$\frac{T}{2}[\text{sinc}(f - f_0)T + \text{sinc}(f + f_0)T]$
14. $W\,\text{sinc}\,Wt$	$\text{rect}\left(\frac{f}{W}\right)$
15. $\begin{cases} 1 - \frac{\lvert t\rvert}{T} & \text{for } \lvert t\rvert \le T \\ 0 & \text{for } \lvert t\rvert > T \end{cases}$	$T\,\text{sinc}^2 fT$
16. $\sum_{m=-\infty}^{\infty}\delta(t - mT_0)$	$\frac{1}{T_0}\sum_{n=-\infty}^{\infty}\delta\left(f - \frac{n}{T_0}\right)$

Note: rect $(f/2W) = 1$ for $-W < f < W$, 0 for $|f| > W$, and sinc $x = (\sin \pi x)/\pi x$.

Často používané vzťahy

$$\cos x\cos y = \tfrac{1}{2}\cos(x + y) + \tfrac{1}{2}\cos(x - y) \quad (D.1)$$

$$\sin x\sin y = -\tfrac{1}{2}\cos(x + y) + \tfrac{1}{2}\cos(x - y) \quad (D.2)$$

$$\sin x\cos y = \tfrac{1}{2}\sin(x + y) + \tfrac{1}{2}\sin(x - y) \quad (D.3)$$

$$\cos x\sin y = \tfrac{1}{2}\sin(x + y) - \tfrac{1}{2}\sin(x - y) \quad (D.4)$$

$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y \quad (D.5)$$

$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y \quad (D.6)$$

$$\cos^2 x = \tfrac{1}{2}(1 + \cos 2x) \quad (D.7)$$

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x) \quad (D.8)$$

$$\sin x\cos x = \tfrac{1}{2}\sin 2x \quad (D.9)$$

$$\sin x + \sin y = 2\sin\tfrac{1}{2}(x + y)\cos\tfrac{1}{2}(x - y) \quad (D.10)$$

$$\sin x - \sin y = 2\cos\tfrac{1}{2}(x + y)\sin\tfrac{1}{2}(x - y) \quad (D.11)$$

$$\cos x + \cos y = 2\cos\tfrac{1}{2}(x + y)\cos\tfrac{1}{2}(x - y) \quad (D.12)$$

$$\cos x - \cos y = -2\sin\tfrac{1}{2}(x + y)\sin\tfrac{1}{2}(x - y) \quad (D.13)$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2j} \quad (D.14)$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2} \quad (D.15)$$

$$\left.\begin{aligned} \int_{-\pi}^{\pi}\sin m\lambda\,d\lambda &= 0 \\ \int_{-\pi}^{\pi}\cos m\lambda\,d\lambda &= 0 \\ \int_{-\pi}^{\pi}\sin m\lambda\cos n\lambda\,d\lambda &= 0 \end{aligned}\right\} \text{where } m \text{ and } n \text{ are any integers} \quad (A.2)$$

$$\left.\begin{aligned} \int_{-\pi}^{\pi}\sin m\lambda\sin n\lambda\,d\lambda &= 0 \\ \int_{-\pi}^{\pi}\cos m\lambda\cos n\lambda\,d\lambda &= 0 \end{aligned}\right\} \text{for } m \neq n \quad (A.3)$$

$$\left.\begin{aligned} \int_{-\pi}^{\pi}(\sin m\lambda)^2\,d\lambda &= \pi \\ \int_{-\pi}^{\pi}(\cos m\lambda)^2\,d\lambda &= \pi \end{aligned}\right\} \text{for } m = n \quad (A.4)$$